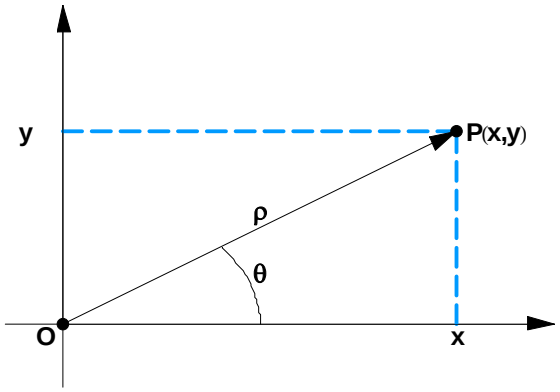
5

REPRESENTATION OF CURVES IN POLAR COORDINATES

5.1. Representation of curves in polar coordinates

Any point P is perfectly defined in a bidimensional system OXY by its cartesian coordinates (x,y). The cartesian coordinates are the orthogonal projections of the point P in the axes. In the same way, any point P is perfectly defined by its polar coordinates (ρ,ϑ); where $\rho>0$ is the distance between P and the origin and the value ϑ is the angle formed by the vector OP and the axis OX in its positive direction.



▼ PolarPlot[ ]

```
? PolarPlot
```

PolarPlot[r, {θ, θ_{min}, θ_{max}}] generates a polar plot of a curve with radius r as a function of angle θ.
PolarPlot[{f_1, f_2, ...}, {θ, θ_{min}, θ_{max}}] makes a polar plot of curves with radius functions f_1, f_2, ... ≫

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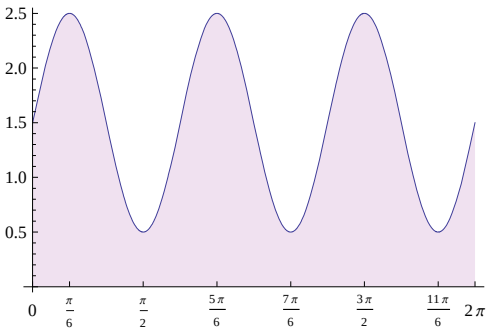
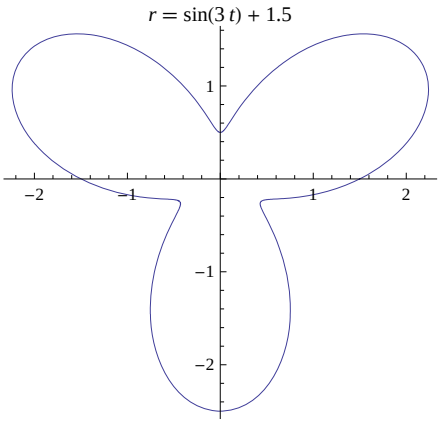
```
Clear["Global`*"]
Plot[1.5 + Sin[3 * t], {t, 0, 2 π}, AxesOrigin → {0, 0},
 Ticks → {{0, π / 6, π / 2 , 5 π / 6, 7 π / 6, 3 π / 2, 11 π / 6, 2 π} , Automatic},
 Filling → Axis, FillingStyle → {LightBlue, LightPurple}]
```




```
PolarPlot[{1.5 + Sin[3 * t]}, {t, 0, 2 π}, PlotLabel → r == 1.5 + Sin[3 * t]]
```



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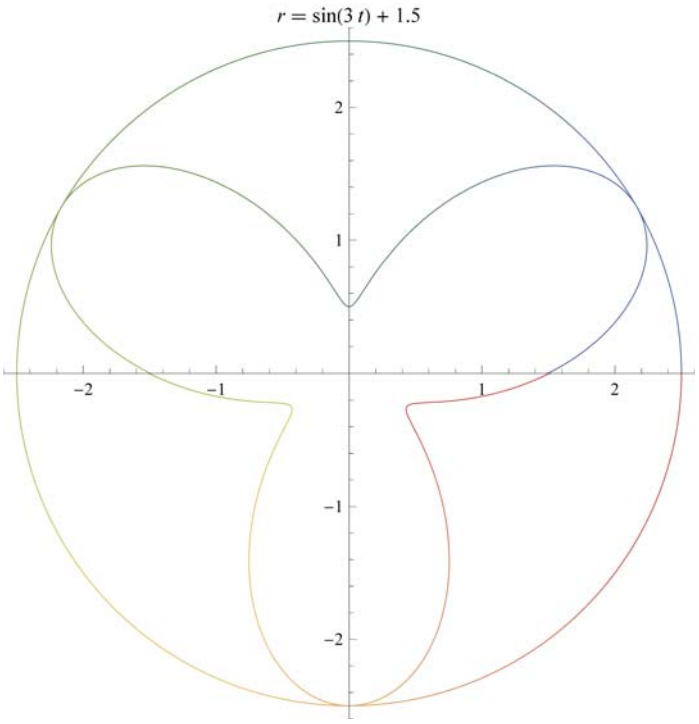
▼ Options of the command PolarPlot[ ]

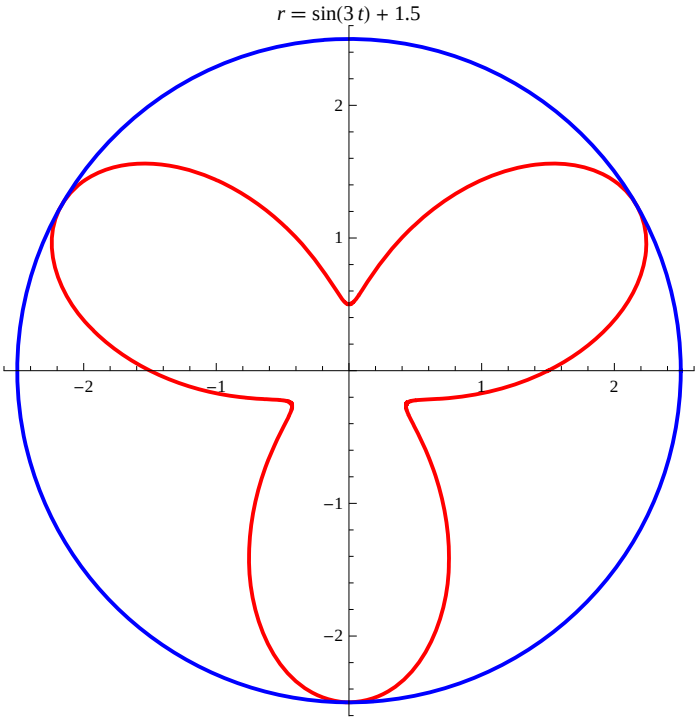
★ ColorFunction

```
PolarPlot[{1.5 + Sin[3 * t], 2.5}, {t, 0, 2 π},
 ColorFunction → "DarkRainbow", PlotLabel → r == 1.5 + Sin[3 * t]]
```



★ PlotStyle

```
PolarPlot[{1.5 + Sin[3 * t], 2.5}, {t, 0, 2 π},
 PlotStyle → {Directive[Red, Thick], Directive[Blue, Thick]},
 PlotLabel → r == 1.5 + Sin[3 * t]]
```



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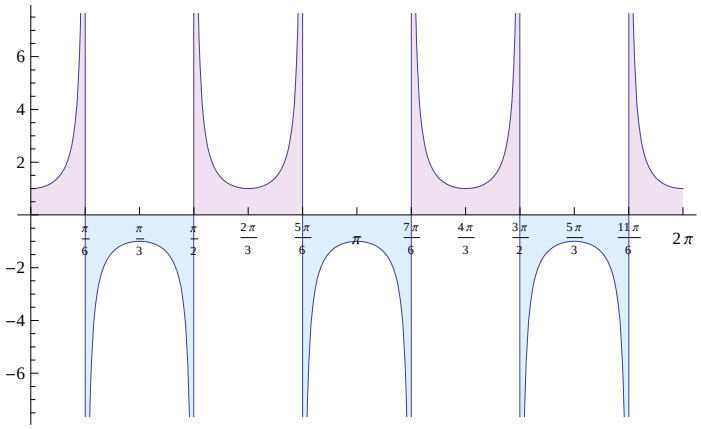
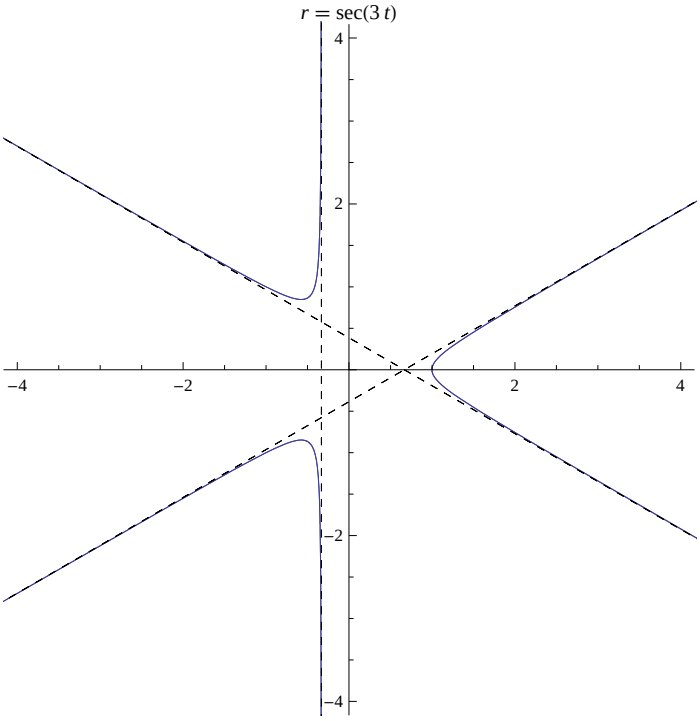
★ Exclusions

```
Plot[1 / Cos[3 t], {t, 0, 2 π}, Filling → Axis,
 FillingStyle → {LightBlue, LightPurple}, Ticks → {Table[k * π / 6, {k, 0, 12}] ,
   Automatic}, AxesOrigin → {0, 0}]
```




```
PolarPlot[1 / Cos[3 t], {t, 0, 2 π}, PlotLabel → r == Sec[3 t],
 Exclusions → {Cos[3 t] == 0}, ExclusionsStyle → Dashed, PlotRange → 4]
```



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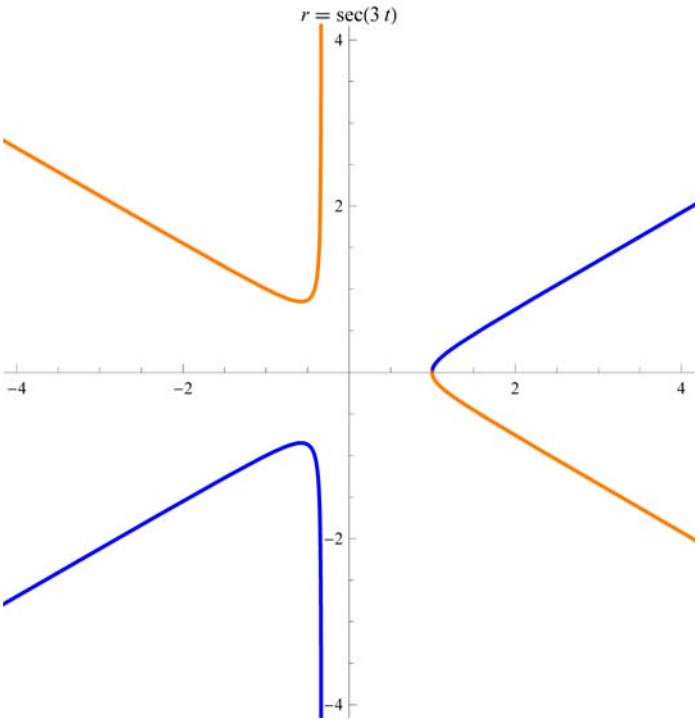
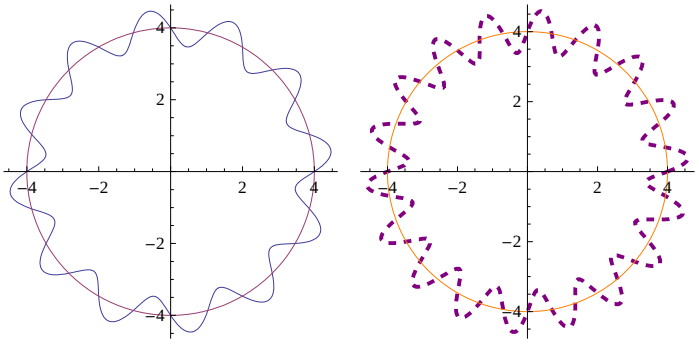
★ Some other style options

```
PolarPlot[1 / Cos[3 t], {t, 0, Pi}, PlotLabel → r == Sec[3 t],
 ColorFunction → Function[{x, y, t, r}, If[ Cos[3 t] < 0, Orange, Blue]],
 PlotStyle → Thick, Exclusions → {Cos[3 t] == 0}, PlotRange → 4]
```



★ Some other options of PolarPlot[ ]

```
g1 = PolarPlot[{4 + 0.5 * Sin[12 * t], 4}, {t, 0, 2 π}];
g2 = PolarPlot[{4 + 0.6 * Sin[18 * t], 4}, {t, 0, 2 π},
   PlotStyle → {Directive[Dashed, Thick, Purple], Orange}]; GraphicsGrid[{{g1, g2}}]
```



5.2. Some functions given in polar form

▼ Circles

★ General equation of a circumference: centre (a,b) and radius c

```
eq = (x - a) ^2 + (y - b) ^2 == c^2
```

$$(-a+x)^2+(-b+y)^2 == c^2$$

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★ Circle 1: Centred in OY, and being (a,b)=(0,b), a=0 and c=b

```
eq1 = eq /. {a → 0, c → b}
```

$$x^2 + (-b + y)^2 == b^2$$

```
x^2 + (-b + y)^2 == b^2
```

$$x^2 + (-b + y)^2 == b^2$$

```
polar1 = eq1 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$$r[t]^2 == 2\, b\, r[t]\, \text{Sin}[t]$$

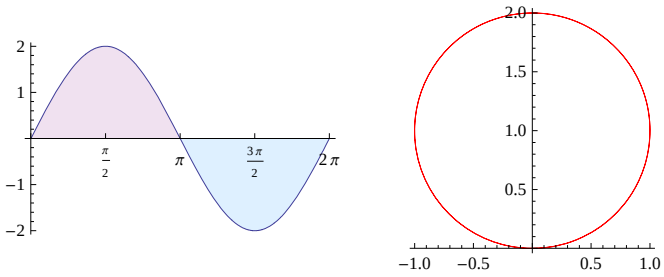
```
Solve[polar1, r[t]]
```

$$\{\{r[t] \to 0\}, \{r[t] \to 2\, b\, \text{Sin}[t]\}\}$$

```
{{r[t] → 0}, {r[t] → 2 b Sin[t]}}
```

$$\{\{r[t] \to 0\}, \{r[t] \to 2\, b\, \text{Sin}[t]\}\}$$

```
circ1[t_, b_] = 2 * b Sin[t];
g1 = Plot[circ1[t, 1], {t, 0, 2 π}, Ticks → {{0, π / 2 , π, 3 π / 2, 2 π} , Automatic},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
c1 = PolarPlot[circ1[t, 1], {t, 0, 2 π}, PlotStyle → Red]; GraphicsGrid[{{g1, c1}}]
```



★ Circle 2: Centred in OX, and being (a,b)=(a,0), b=0 and c=a

```
eq2 = eq /. {b → 0, c → a}
```

$$(-a + x)^2 + y^2 == a^2$$

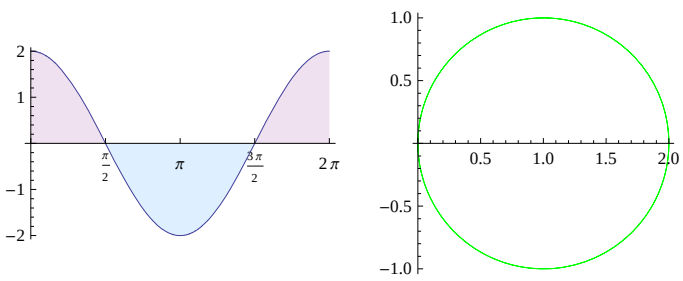
```
polar2 = eq2 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$$2\, a\, \text{Cos}[t]\, r[t] == r[t]^2$$

```
Solve[polar2, r[t]]
```

$$\{\{r[t] \to 0\}, \{r[t] \to 2\, a\, \text{Cos}[t]\}\}$$

```
circ2[t_, a_] = 2 * a Cos[t];
g2 = Plot[circ2[t, 1], {t, 0, 2 π}, Ticks → {{0, π / 2 , π, 3 π / 2, 2 π} , Automatic},
   Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
c2 = PolarPlot[circ2[t, 1], {t, 0, 2 π}, PlotStyle → Green];
GraphicsGrid[{{g2, c2}}]
```



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★ Circle 3: Centred in the origin, and being (a,b)=(0,0), a=0 and b=0

```
eq3 = eq /. {a → 0, b → 0}
```

$$x^2 + y^2 == c^2$$

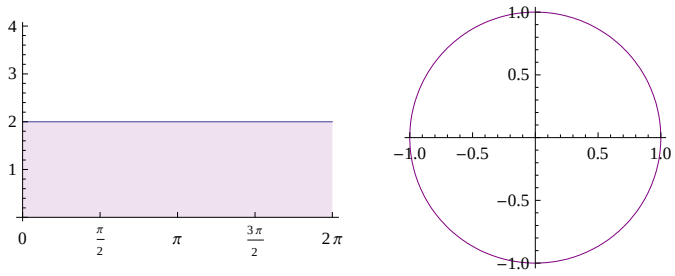
```
polar3 = eq3 /. {x → r[t] * Cos[t], y → r[t] * Sin[t]} // Simplify
```

$$c^2 == r[t]^2$$

```
Solve[polar3, r[t]]
```

$$\{\{r[t] \to -c\}, \{r[t] \to c\}\}$$

```
circ3[t_, a_] = a;
g3 = Plot[circ3[t, 2] , {t, 0, 2 π}, Ticks → {{0, π / 2 , π, 3 π / 2, 2 π} , Automatic},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
c3 = PolarPlot[circ3[t, 1], {t, 0, 2 π}, PlotStyle → Purple];
GraphicsGrid[{{g3, c3}}]
```



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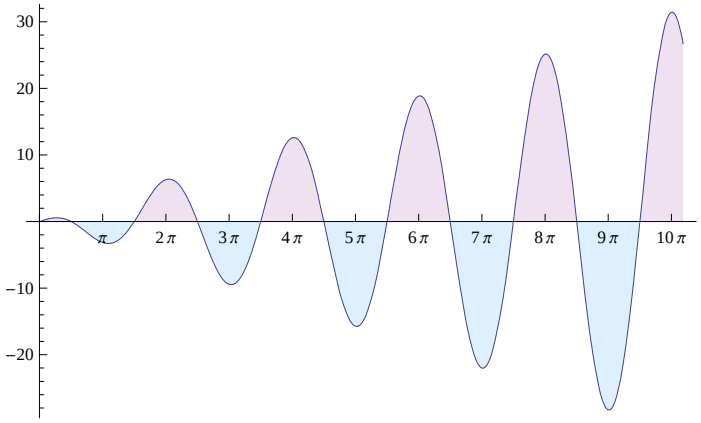
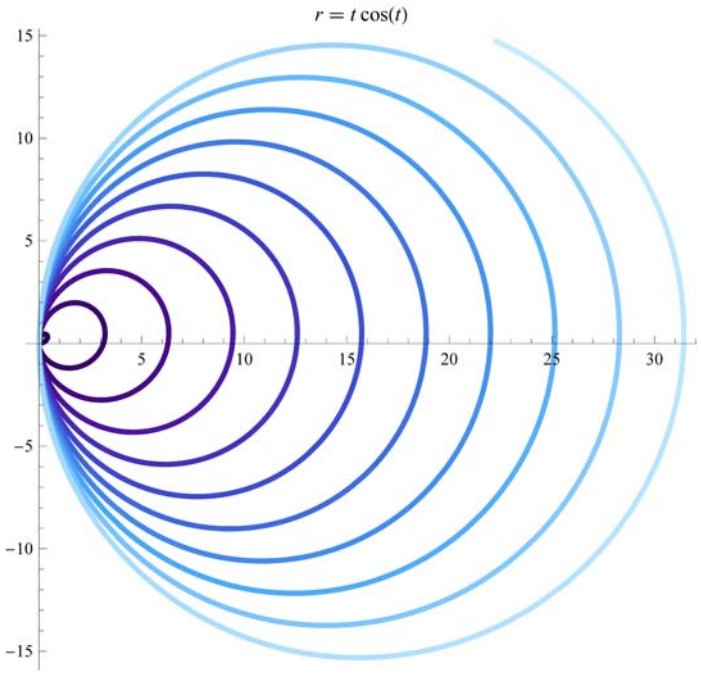
★ SPIRAL OF CIRCLES

```
g3 = Plot[t Cos[t], {t, 0, 32}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {Table[k * π , {k, 0, 10}] , Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[ t Cos[t], {t, 0, 32}, ColorFunction → "DeepSeaColors",
  PlotStyle → Thickness[0.008], PlotLabel → r == t Cos[t]]
```





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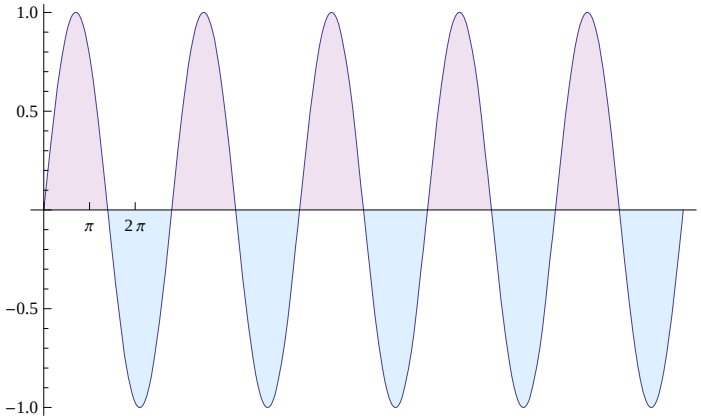
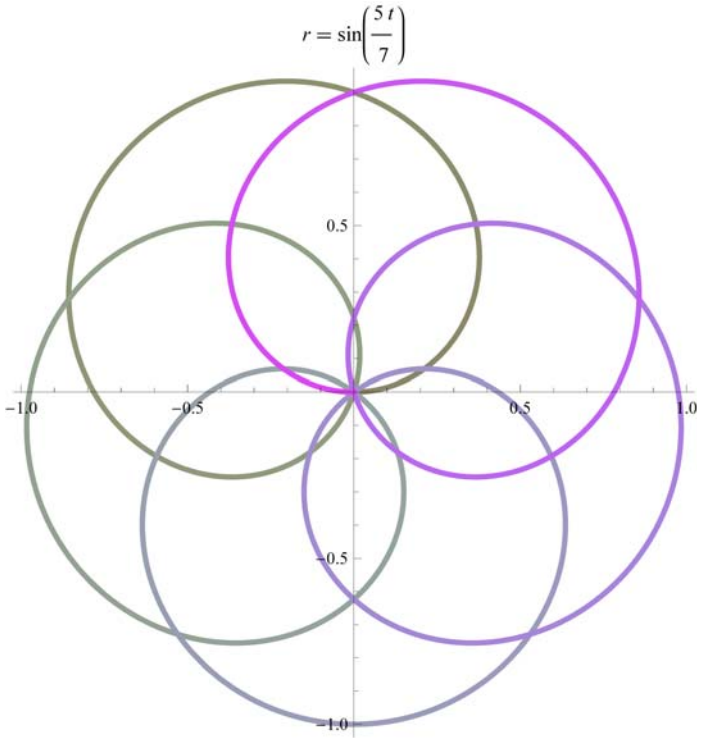
★ PSEUDOCIRCLES

```
g3 = Plot[Sin[5 t / 7], {t, 0, 14 π},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[ Sin[5 t / 7], {t, 0, 14 π}, ColorFunction → "AuroraColors",
  PlotStyle → Thickness[0.008], PlotLabel → r == Sin[5 t / 7]]
```





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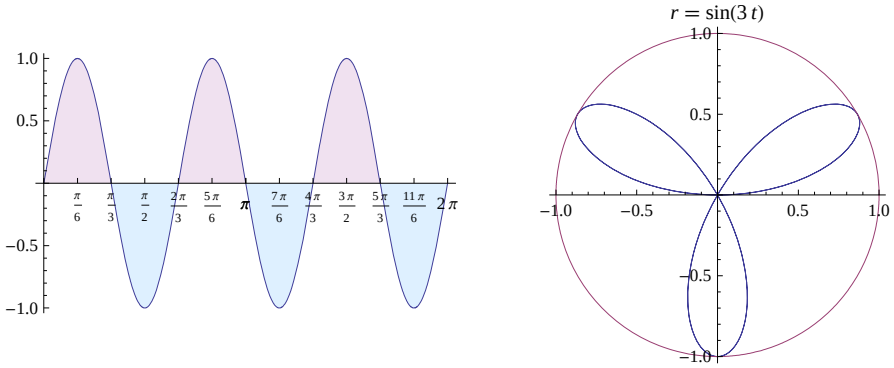
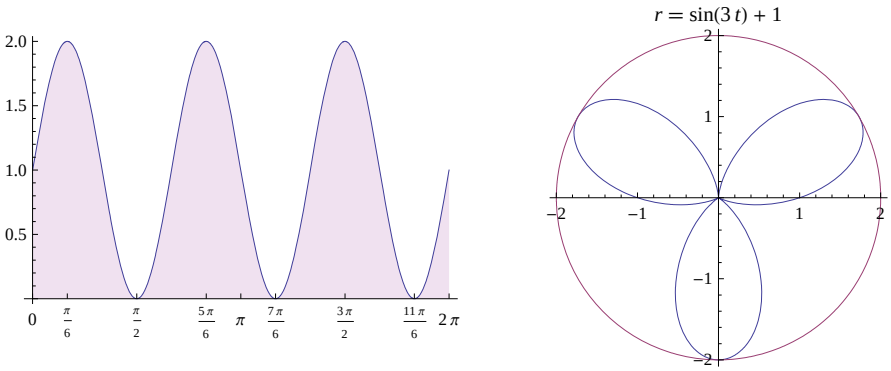
▼ Roses

★ 3-leaves rose

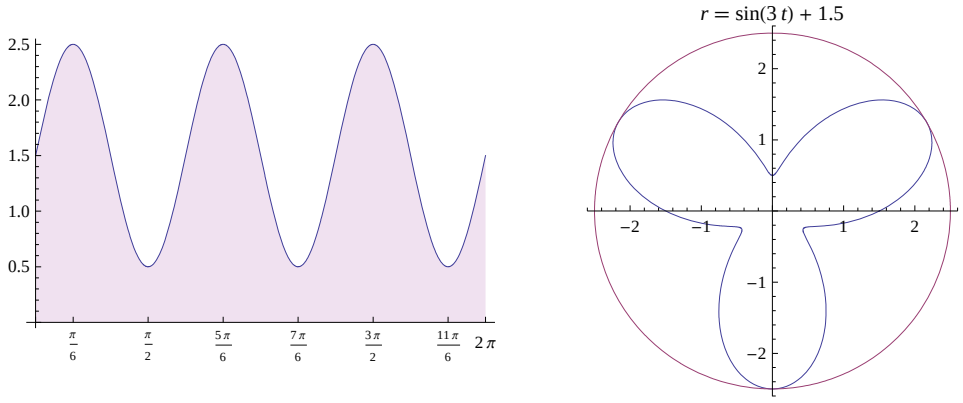
```
g1 = Plot[Sin[3 * t], {t, 0, 2 π}, Ticks → {{0, π / 6, π / 3 , 2 π / 3, π / 2 ,
       π, 5 π / 6, π , 7 π / 6, 4 π / 3, 3 π / 2, 5 π / 3, 11 π / 6, 2 π} , Automatic},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
g2 = PolarPlot[{Sin[3 * t], 1}, {t, 0, 2 π}, PlotLabel → r == Sin[3 * t]];
GraphicsGrid[{{g1, g2}}]
```



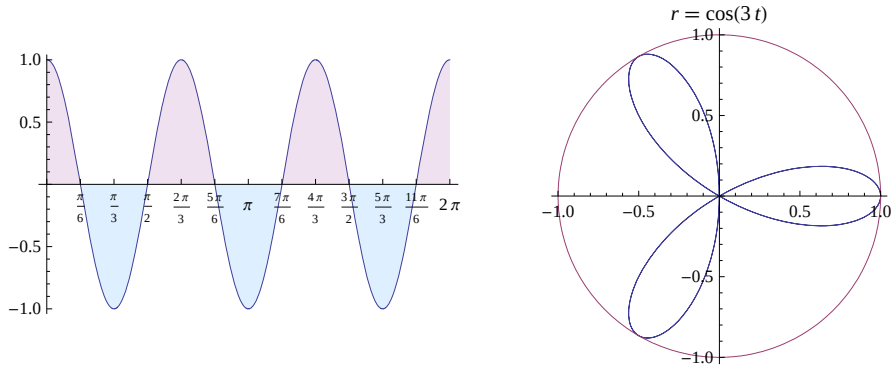

```
g1 = Plot[1 + Sin[3 * t], {t, 0, 2 π},
    Ticks → {{0, π / 6, π / 2 , 5 π / 6, π, 7 π / 6, 3 π / 2, 11 π / 6, 2 π} , Automatic},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
g2 = PolarPlot[{1 + Sin[3 * t], 2}, {t, 0, 2 π}, PlotLabel → r == 1 + Sin[3 * t]];
GraphicsGrid[{{g1, g2}}]
```



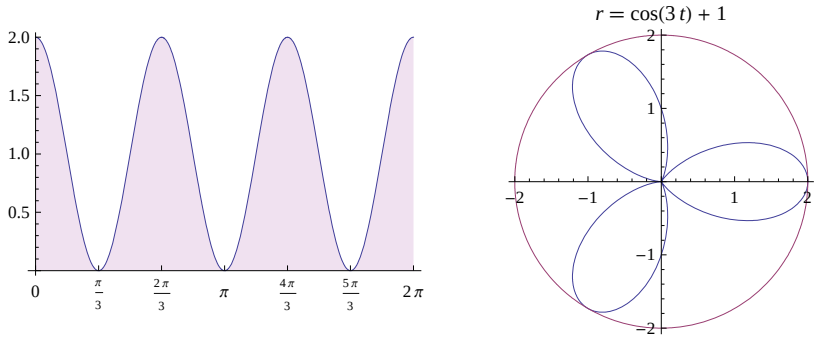
```
g1 = Plot[1.5 + Sin[3 * t], {t, 0, 2 π}, AxesOrigin → {0, 0},
    Ticks → {{0, π / 6, π / 2 , 5 π / 6, 7 π / 6, 3 π / 2, 11 π / 6, 2 π} , Automatic},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple}];
g2 = PolarPlot[{1.5 + Sin[3 * t], 2.5}, {t, 0, 2 π}, PlotLabel → r == 1.5 + Sin[3 * t]];
GraphicsGrid[{{g1, g2}}]
```




```
g1 = Plot[Cos[3 * t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 6, π / 3 , π / 2 , 2 π / 3, 5 π / 6, π , 7 π / 6, 4 π / 3,
       3 π / 2, 5 π / 3, 11 π / 6, 2 π} , Automatic}, AxesOrigin → {0, 0}];
g2 = PolarPlot[{Cos[3 * t], 1}, {t, 0, 2 π}, PlotLabel → r == Cos[3 * t]];
GraphicsGrid[{{g1, g2}}]
```




```
g1 = Plot[1 + Cos[3 * t], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 3 , 2 π / 3, π , 4 π / 3, 5 π / 3, 2 π} , Automatic}, AxesOrigin → {0, 0}];
g2 = PolarPlot[{1 + Cos[3 * t], 2}, {t, 0, 2 π}, PlotLabel → r == 1 + Cos[3 * t]];
GraphicsGrid[{{g1, g2}}]
```



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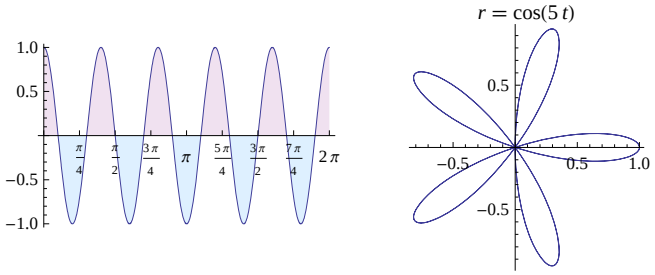
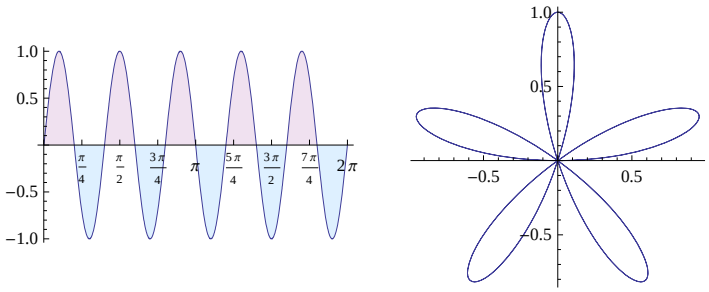
★ 5-leaves rose

```
g1 = Plot[Cos[5 * t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 4 , π / 2, 3 π / 4, π , 5 π / 4 , 3 π / 2, 7 π / 4, 2 π} , Automatic},
    AxesOrigin → {0, 0}];

g2 = PolarPlot[Cos[5 * t], {t, 0, 2 π}, PlotLabel → r == Cos[5 * t]];

GraphicsGrid[{{g1, g2}}]
```



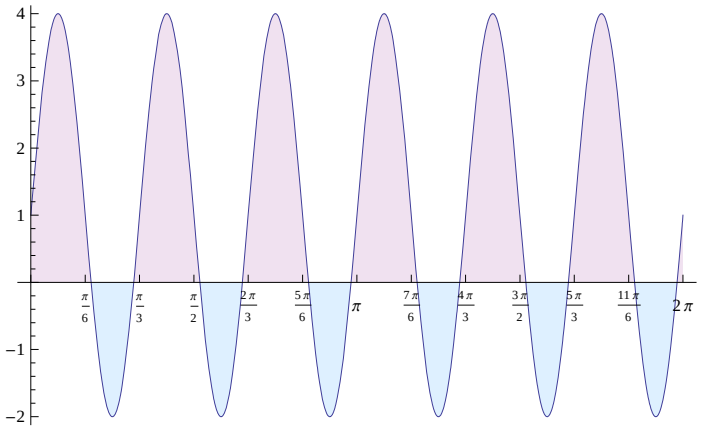

```
g1 = Plot[Sin[5 * t], {t, 0, 2 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 4 , π / 2, 3 π / 4, π , 5 π / 4 , 3 π / 2, 7 π / 4, 2 π} , Automatic},
    AxesOrigin → {0, 0}];

g2 = PolarPlot[Sin[5 * t], {t, 0, 2 Pi}];

GraphicsGrid[{{g1, g2}}]
```



▼ Some other roses

★ Example 1

```
g3 = Plot[1 + 3 Sin[6 t], {t, 0, 2 π}, Filling → Axis,
  Ticks → {Table[k * π / 6, {k, 0, 12}] , Automatic},
  FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π , 2 π} , Automatic}, AxesOrigin → {0, 0}]
```



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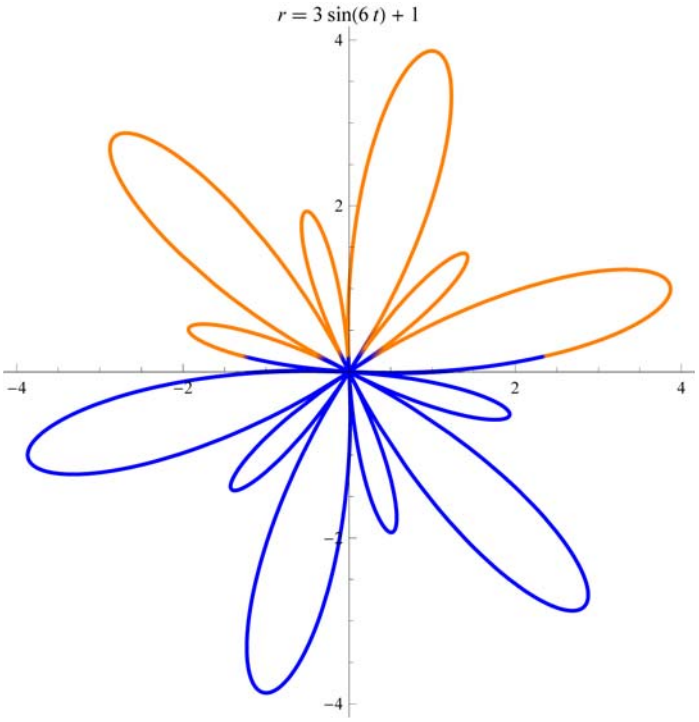
```
PolarPlot[1 + 3 Sin[6 t], {t, 0, 2 Pi},
 ColorFunction → Function[{x, t}, If[Pi / 6 < t < Pi / 3, Orange, Blue]],
 PlotRange → {{-4, 4}, {-4, 4}}, PlotStyle → Thick, PlotLabel → r == 1 + 3 Sin[6 t]]
```



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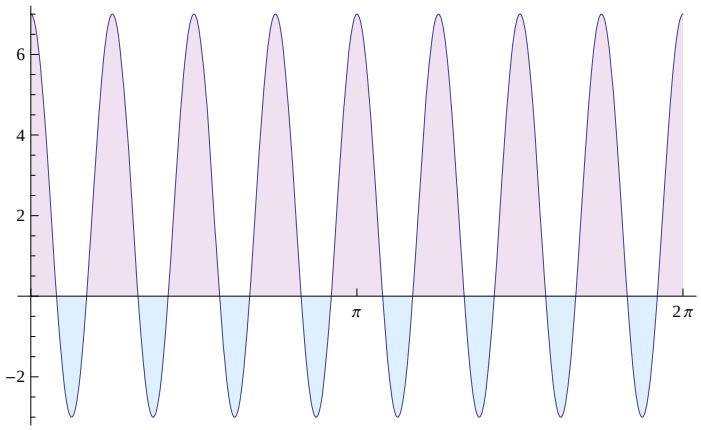
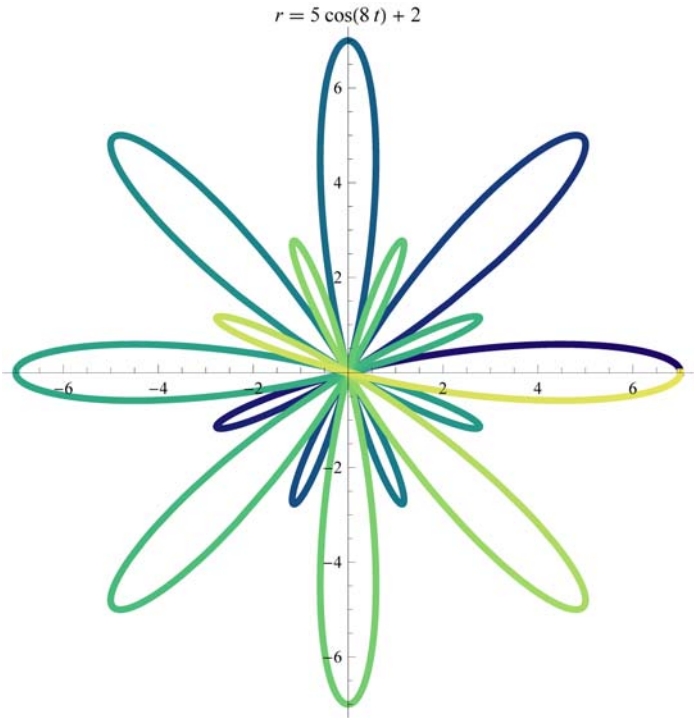
★ Example 2

```
g3 = Plot[2 + 5 Cos[8 t], {t, 0, 2 π},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[2 + 5 Cos[8 t], {t, 0, 2 π}, PlotLabel → r == 2 + 5 Cos[8 t],
  ColorFunction → "BlueGreenYellow", PlotStyle → Thickness[0.01]]
```





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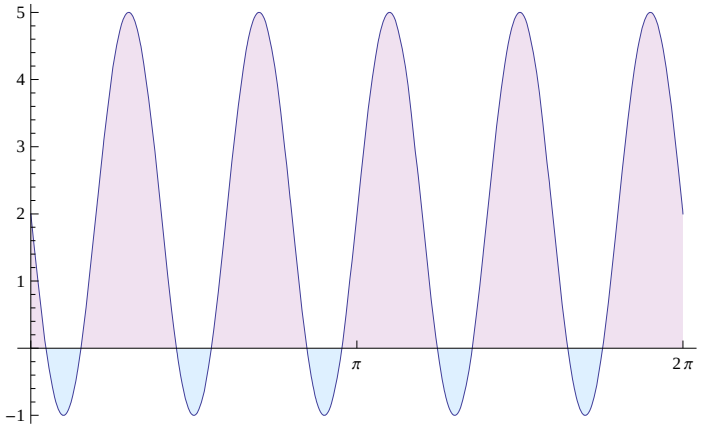
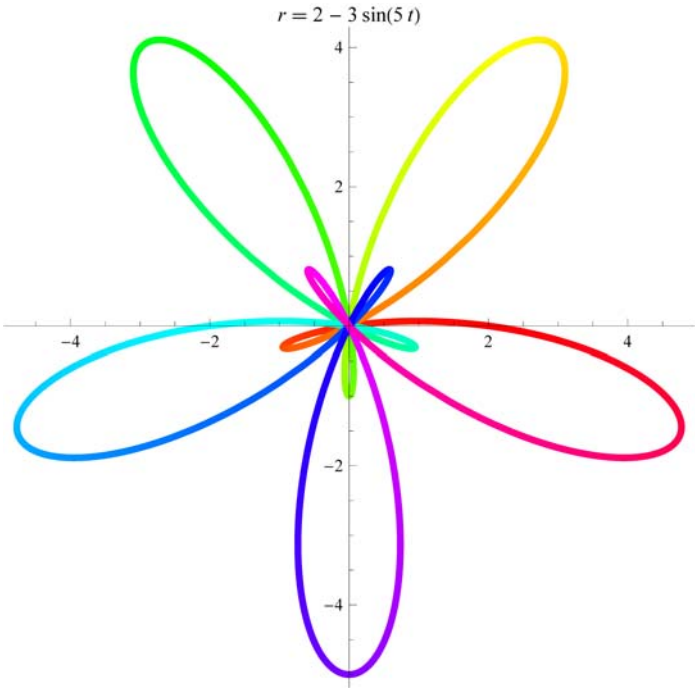
★ Example 3

```
g3 = Plot[2 - 3 Sin[5 t], {t, 0, 2 π},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[2 - 3 Sin[5 t], {t, 0, 2 π}, PlotLabel → r == 2 - 3 Sin[5 t],
  ColorFunction → Function[{x, y, z}, Hue[z]], PlotStyle → Thickness[0.01]]
```





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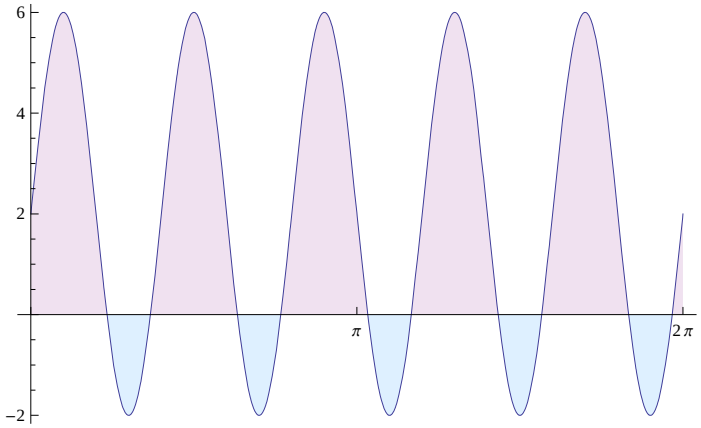
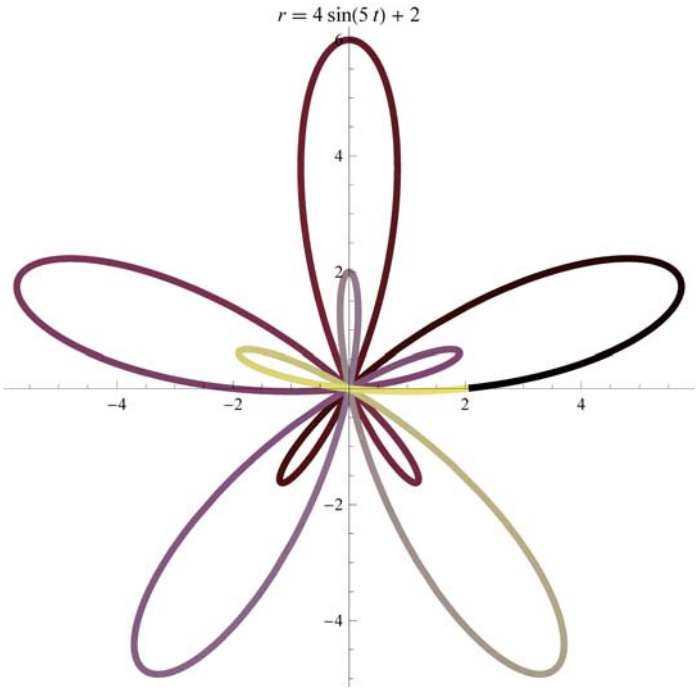
★ Example 4

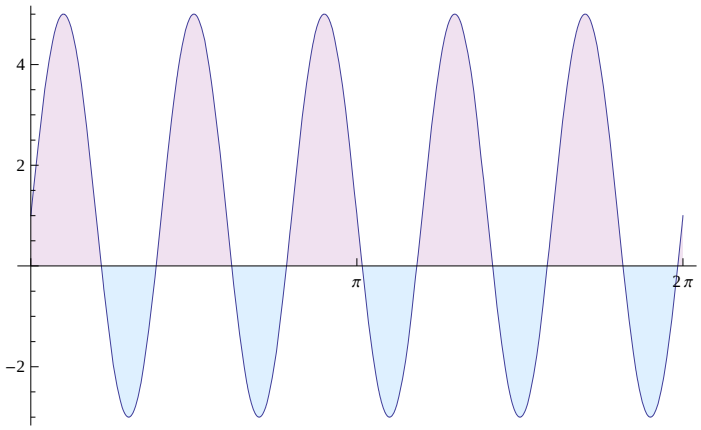
```
g3 = Plot[2 + 4 Sin[5 t], {t, 0, 2 π},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[2 + 4 Sin[5 t], {t, 0, 2 π}, ColorFunction → "PlumColors",
  PlotStyle → Thickness[0.01], PlotLabel → r == 2 + 4 Sin[5 t]]
```

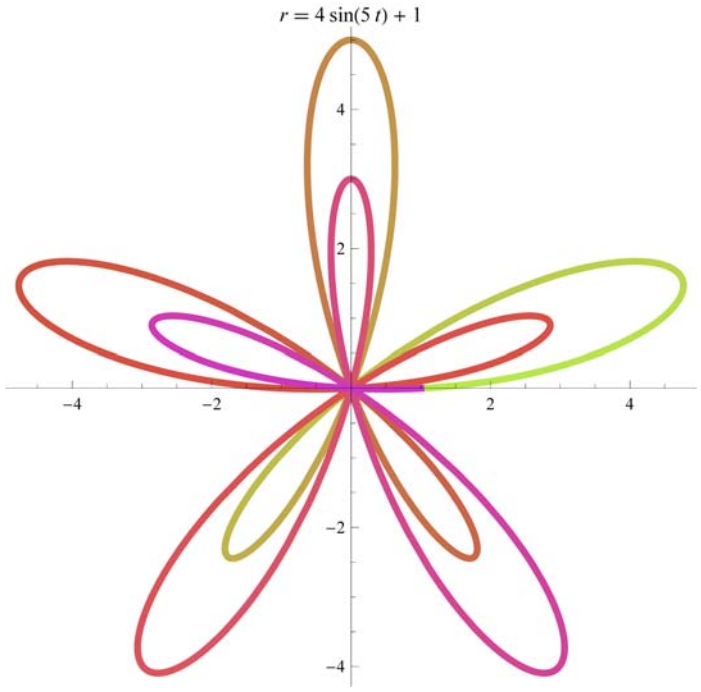




★ Example 5

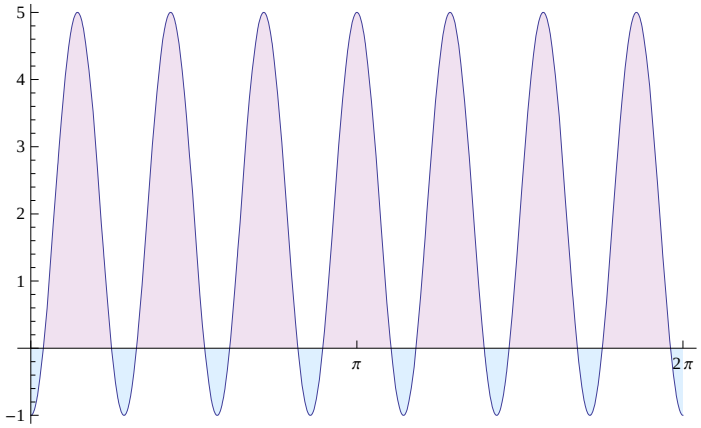
```
g3 = Plot[1 + 4 Sin[5 t], {t, 0, 2 π},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[1 + 4 Sin[5 t], {t, 0, 2 π}, ColorFunction → "NeonColors",
  PlotStyle → Thickness[0.01], PlotLabel → r == 1 + 4 Sin[5 t]]
```

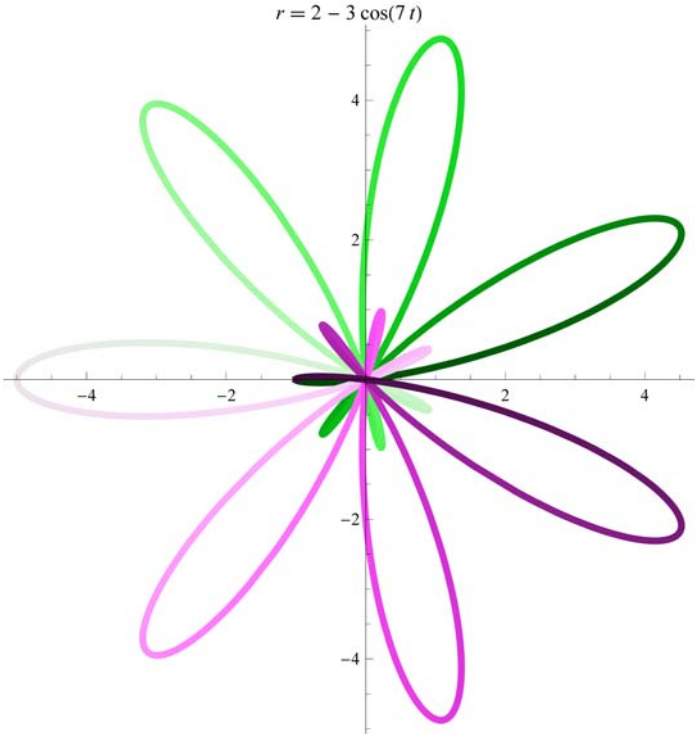




★ Example 6

```
g3 = Plot[2 - 3 Cos[7 t], {t, 0, 2 π},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[2 - 3 Cos[7 t], {t, 0, 2 π}, ColorFunction → "GreenPinkTones",
  PlotStyle → Thickness[0.01], PlotLabel → r == 2 - 3 Cos[7 t]]
```





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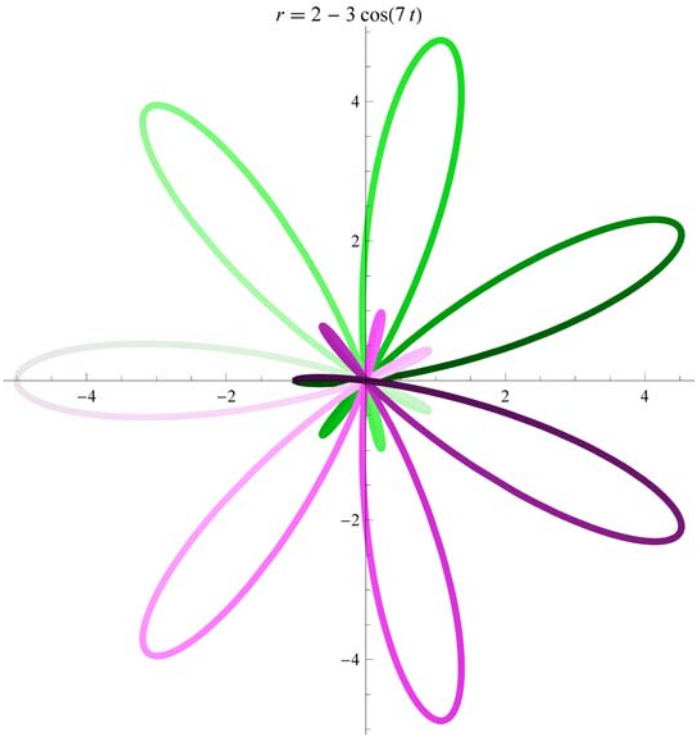
```
PolarPlot[ 2 - 3 Cos[7 t], {t, 0, 2 π}, ColorFunction -> "GreenPinkTones",
 PlotStyle -> Thickness[0.01], PlotLabel -> r == 2 - 3 Cos[7 t]]
```



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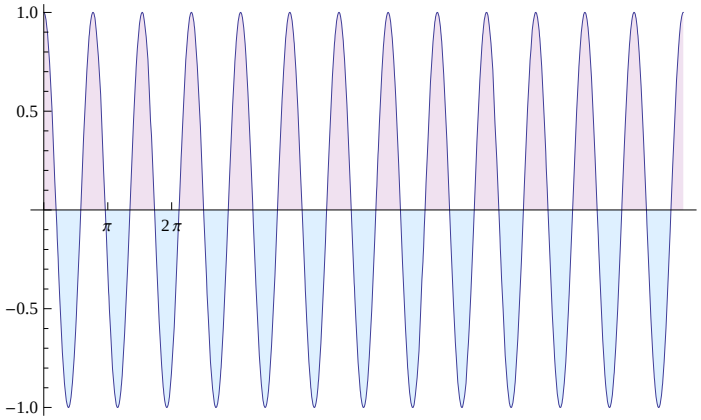
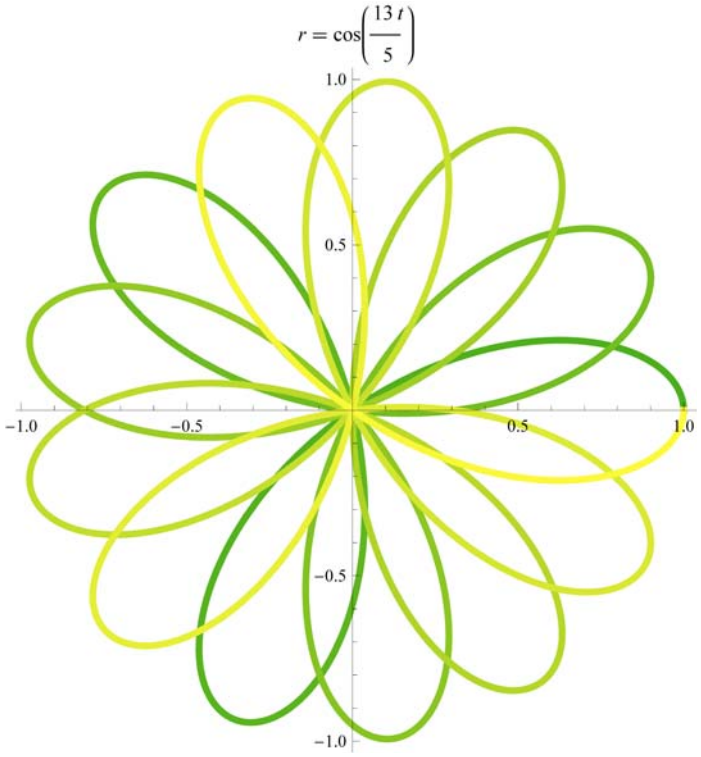
★ Example 7

```
g3 = Plot[Cos[13 t / 5], {t, 0, 10 π},
  Filling → Axis, FillingStyle → {LightBlue, LightPurple},
  Ticks → {{0, π, 2 π}, Automatic}, AxesOrigin → {0, 0}]
c3 = PolarPlot[ Cos[13 t / 5], {t, 0, 10 π}, ColorFunction → "AvocadoColors",
  PlotStyle → Thickness[0.01], PlotLabel → r == Cos[13 t / 5]]
```





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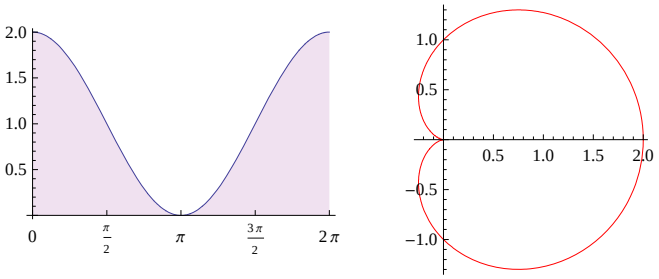
▼ Cardioid

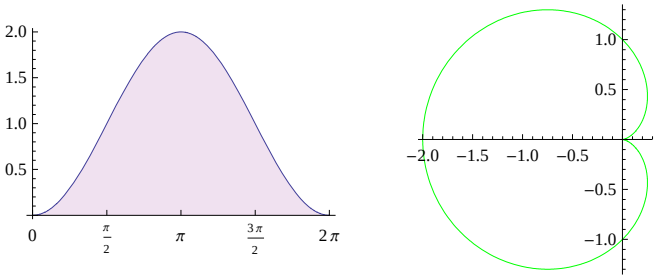
★ Cardioid 1

```
cardioid1[t_, a_] = a (1 + Cos[t]);
g1 = Plot[cardioid1[t, 1], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 2, π , 3 π / 2, 2 π} , Automatic}, AxesOrigin → {0, 0}];
kar1 = PolarPlot[cardioid1[t, 1], {t, 0, 2 π}, PlotStyle → Red]; GraphicsGrid[{{g1, kar1}}]
```



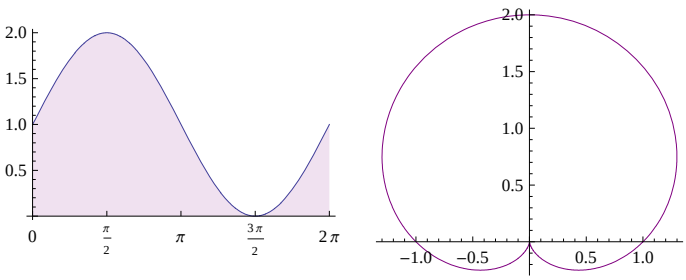
★ Cardioid 2

```
cardioid2[t_, a_] = a (1 - Cos[t]);
g2 = Plot[cardioid2[t, 1], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 2, π , 3 π / 2, 2 π} , Automatic}, AxesOrigin → {0, 0}];
kar2 = PolarPlot[cardioid2[t, 1], {t, 0, 2 π}, PlotStyle → Green];
GraphicsGrid[{{g2, kar2}}]
```



★ Cardioid 3

```
cardioid3[t_, a_] = a (1 + Sin[t]);
g3 = Plot[cardioid3[t, 1], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 2, π , 3 π / 2, 2 π} , Automatic}, AxesOrigin → {0, 0}];
kar3 = PolarPlot[cardioid3[t, 1], {t, 0, 2 π}, PlotStyle → Purple];
GraphicsGrid[{{g3, kar3}}]
```



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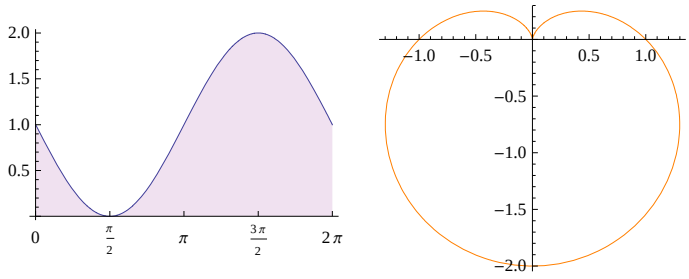
★ Cardioid 4

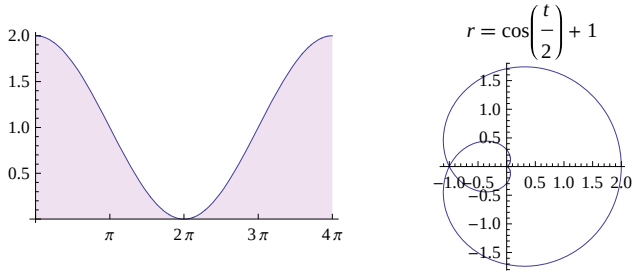
```
cardioid4[t_, a_] = a (1 - Sin[t]);
g4 = Plot[cardioid4[t, 1], {t, 0, 2 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π / 2, π , 3 π / 2, 2 π} , Automatic}, AxesOrigin → {0, 0}];
kar4 = PolarPlot[cardioid4[t, 1], {t, 0, 2 π}, PlotStyle → Orange];
GraphicsGrid[{{g4, kar4}}]
```



▼ Pseudocardioid

★ Example 1

```
g1 = Plot[1 + Cos[t / 2], {t, 0, 4 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π , 2 π, 3 π, 4 π} , Automatic}, AxesOrigin → {0, 0}];
c1 = PolarPlot[1 + Cos[t / 2], {t, 0, 4 π}, PlotLabel → r == 1 + Cos[t / 2]];
GraphicsGrid[{{g1, c1}}]
```



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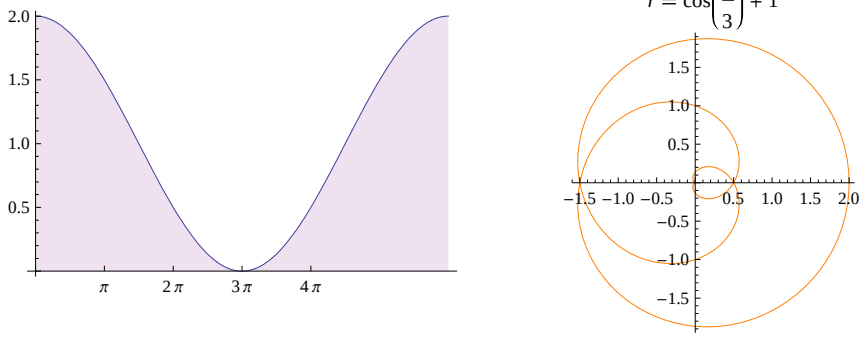
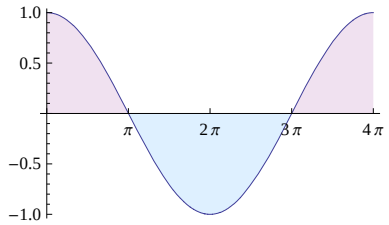
★ Example 2

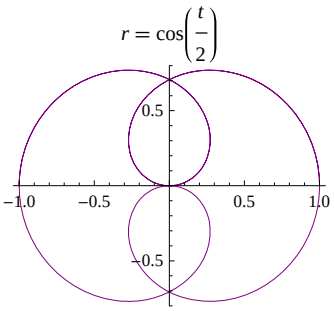
```
g2 = Plot[1 + Cos[t / 3], {t, 0, 6 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π, 2 π, 3 π, 4 π}, Automatic}, AxesOrigin → {0, 0}];
c2 = PolarPlot[1 + Cos[t / 3], {t, 0, 6 π}, PlotLabel → r == 1 + Cos[t / 3],
    PlotStyle → Orange];
GraphicsGrid[{{g2, c2}}]
```



★ Example 3

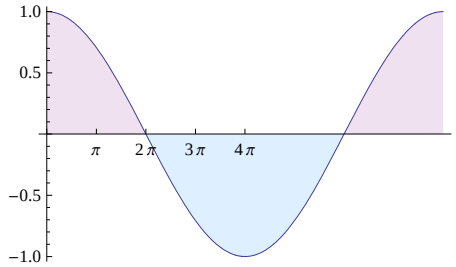
```
g3 = Plot[ Cos[t / 2], {t, 0, 4 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π, 2 π, 3 π, 4 π}, Automatic}, AxesOrigin → {0, 0}];
c3 = PolarPlot[ Cos[t / 2], {t, 0, 6 π}, PlotLabel → r == Cos[t / 2], PlotStyle → Purple];
GraphicsGrid[{{g3, c3}}]
```

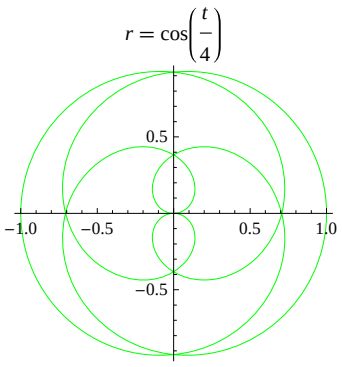




★ Example 4

```
g4 = Plot[ Cos[t / 4], {t, 0, 8 π}, Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, π, 2 π, 3 π, 4 π}, Automatic}, AxesOrigin → {0, 0}];
c4 = PolarPlot[ Cos[t / 4], {t, 0, 8 π}, PlotLabel → r == Cos[t / 4], PlotStyle → Green];
GraphicsGrid[{{g4, c4}}]
```





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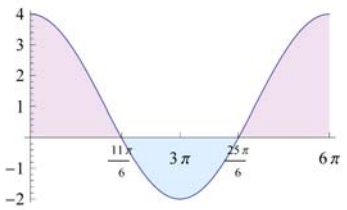
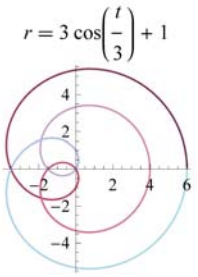
★ **Example 5**

```
g5 = Plot[1 + 3 Cos[ t / 3], {t, 0, 6 π},
    Filling → Axis, FillingStyle → {LightBlue, LightPurple},
    Ticks → {{0, 11 π / 6 , 25 π / 6 , 3 π , 6 π} , Automatic}, AxesOrigin → {0, 0}];
c5 = PolarPlot[ 1 + 5 Cos[ t / 3], {t, 0, 6 π}, ColorFunction → "CandyColors",
    PlotStyle → Thickness[0.01], PlotLabel → r == 1 + 3 Cos[ t / 3]];
GraphicsGrid[{{g5, c5}}]
```





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