1

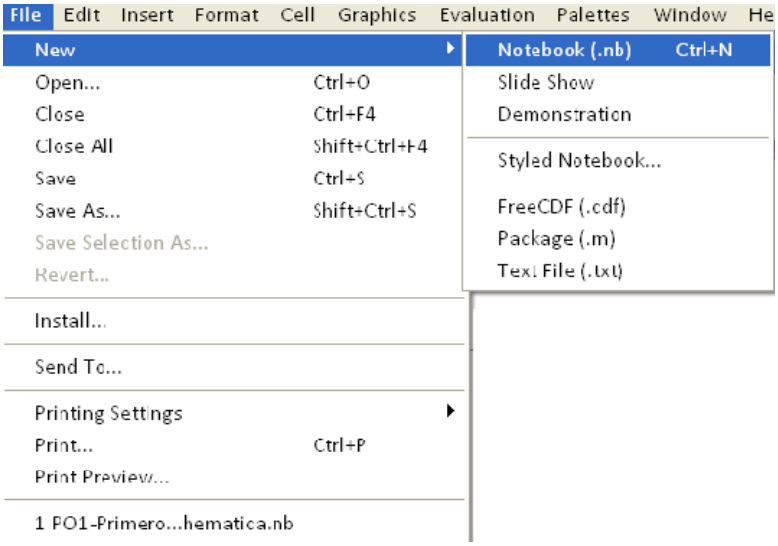
FIRST STEPS WITH "*MATHEMATICA*"

1.1 Creating a document (Notebook)

▼ Starting and finishing a session

★ File// /New//Notebook

These are the steps that have to be followed to create a document:



★ File// Open

It is used to open a document that is already created.

★ File// //Save//Save as

It is used to save a document.

★ File// Printing Settings

It is used to insert headers and footers.

★ File// Print

It is used to print a document.

▼ Writing in a document

★ Palettes// WrittingAssistant-Palets//BasicMathAssistant

We can use palettes to make easier writing in a document, making plots, changing the format of the cells, etc.

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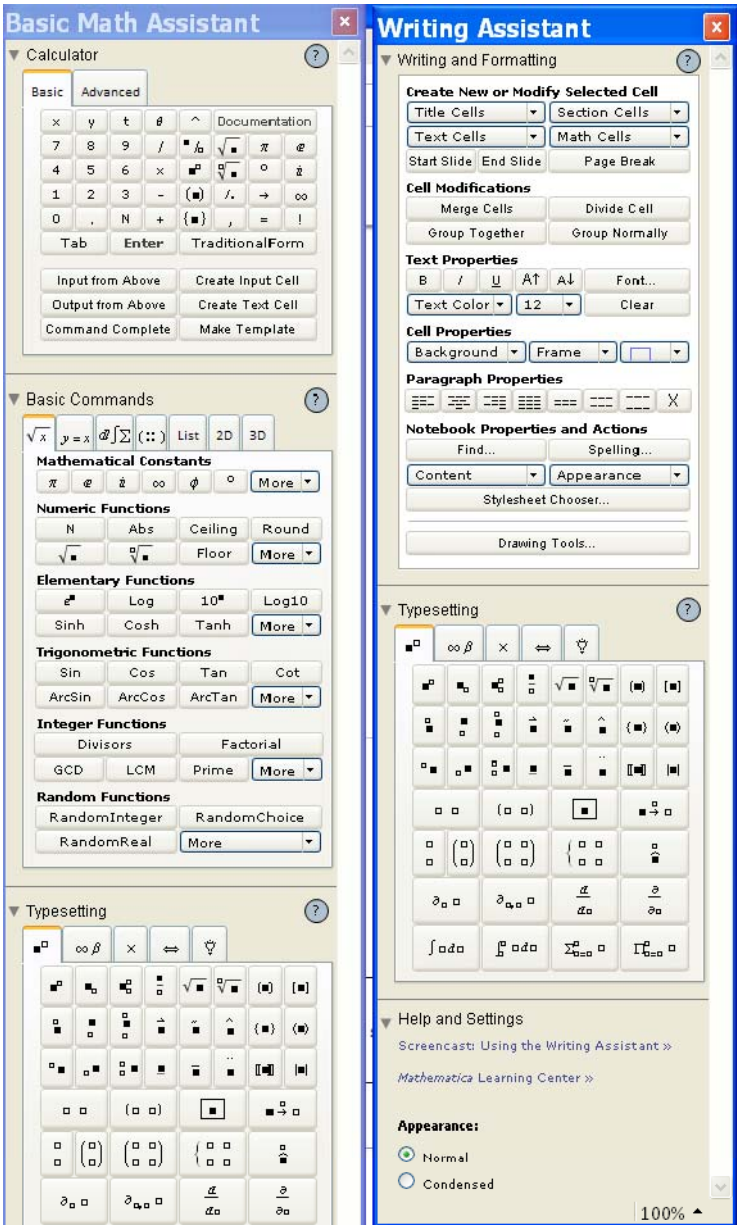

★ Format// Style//

It is possible to give a different style to each of the cells.

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1.2. What is *MATHEMATICA?*

▼ An interactive program

This is a section.

★ Input/Output

This is a subsection.

```
In[2]:= 1 + 1
```

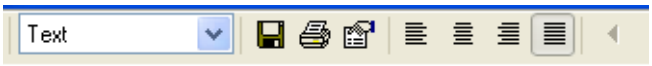
This is an "input".

```
Out[2]= 2
```

This is an "output".

★ Window// Show Toolbar

Using this toolbar, it is easy to access the possibilities to change styles: save, print, and so on.



▼ Information and Help

★ Help// Find Select Function

★ Help//Function Navigator/Mathematics and Algorithms or Documentation Center

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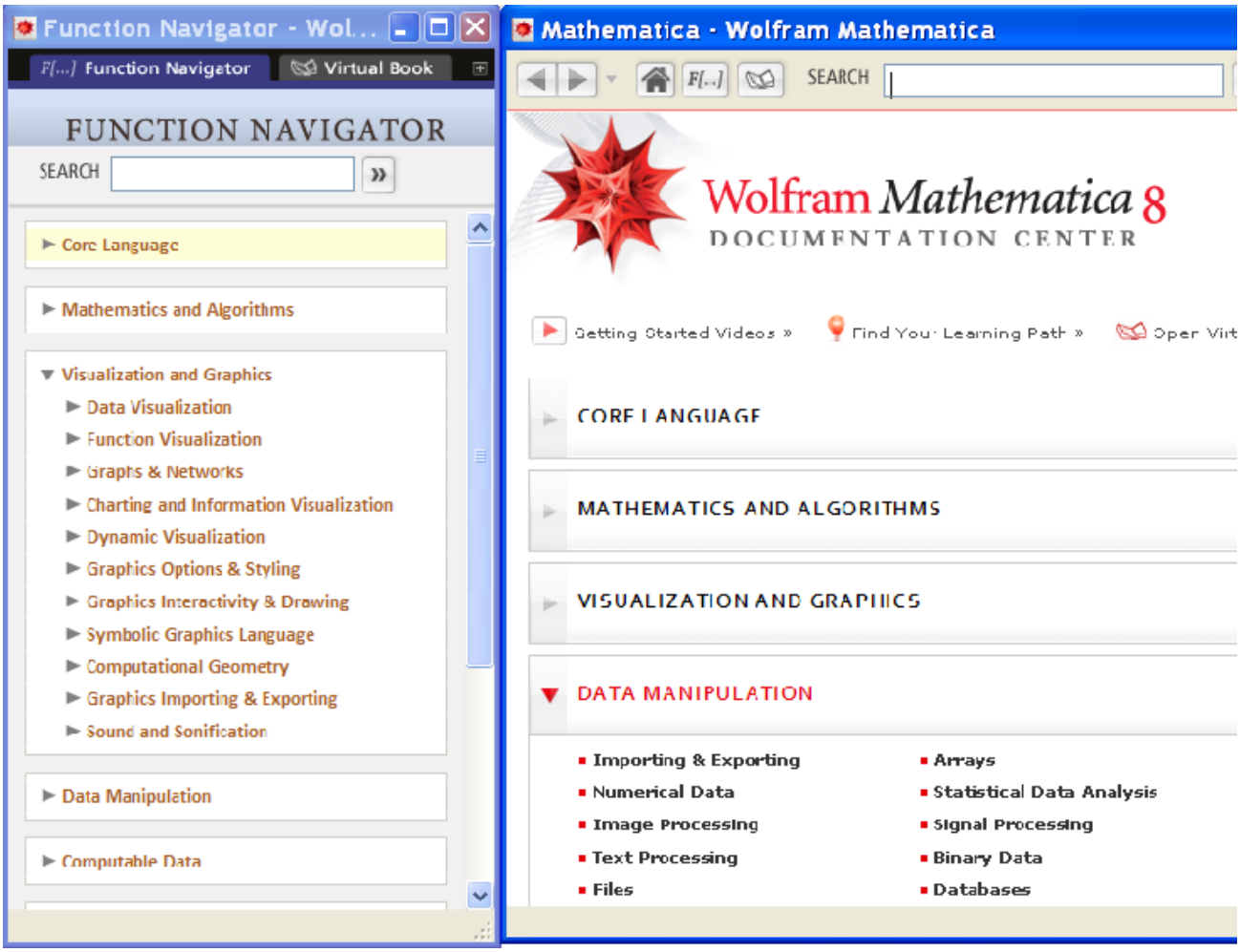

1.2. What is *MATHEMATICA?*

▼ An interactive program

★ Input/Output

When a line is executed the answer is immediate:

```
In[1]:= 1 + 1

Out[1]= 2

In[2]:= %

Out[2]= 2

In[3]:= %%

Out[3]= 2

In[4]:= %1

Out[4]= 2
```


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▼ *Mathematica* is a powerful tool for symbolic calculations

When we are using symbols, *Mathematica* uses the expressions in algebraic form and it shows the results in symbolic form.

★ Basic arithmetic operations

```
{5 - 1, 2 * 5, 2 × 5, 25, 2^5, 13 / 3, √9}
```

$$\left\{4,\ 10,\ 10,\ 25,\ 32,\ \frac{13}{3},\ 3\right\}$$

Parenthesis are used to set priorities in operations

```
(4 + 3 / 2 - 7) ^ 2 / 3
```

$$\frac{3}{4}$$

```
((4 + 3) / (2 - 7)) ^ (2 / 3)
```

$$\left(-\frac{7}{5}\right)^{2/3}$$

```
(4 + 3 / 2 - 7) ^ (2 / 3)
```

$$\left(-\frac{3}{2}\right)^{2/3}$$

```
((4 + 3 / 2 - 7) ^ 2) ^ (1 / 3)
```

$$\left(\frac{3}{2}\right)^{2/3}$$

★ Relational operators

```
2 ≥ 4
```

```
False
```

```
3 == 3
```

```
True
```

★ Derivatives

The derivative of a one variable function

```
D[Log[x], x]
```

$$\frac{1}{x}$$

★ The sum

```
Sum[1 / 2^n, {n, 1, 5}]
```

$$\frac{31}{32}$$

```
Sum[a^n, {n, 1, 5}]
```

$$a + a^2 + a^3 + a^4 + a^5$$

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★ Integration

```
Integrate[Sin[x]^2,x]
```

$$\frac{x}{2}-\frac{1}{4}\text{Sin}[2\,x]$$

```
Integrate[Sin[x]^2,{x,0,Pi}]
```

$$\frac{\pi}{2}$$

★ Solution of algebraic equations

```
Solve[x^5+2==0,x]
```

$$\left\{\left\{x\to(-2)^{1/5}\right\},\left\{x\to-2^{1/5}\right\},\left\{x\to-(-1)^{2/5}\,2^{1/5}\right\},\left\{x\to(-1)^{3/5}\,2^{1/5}\right\},\left\{x\to-(-1)^{4/5}\,2^{1/5}\right\}\right\}$$

★ Limit calculations

```
Limit[x + 4, x → 2]
```

6

```
Limit[ⅇ^x, x → ∞]
```

∞

★ Predetermined functions and constants

See: Function Navigator/Mathematics and Algorithms

The angle of the trigonometric functions is given always in radians

```
{Pi,E,ⅇ^0,π/2,I,I^2,ⅈ^2}
```

$$\left\{\pi,\ e,\ 1,\ \frac{\pi}{2},\ i,\ -1,\ -1\right\}$$

```
{Sin[π/3], Sin[60 Degree], Log[E], Exp[1], Exp[Log[x]]}
```

$$\left\{\text{Sin}\left[\frac{\pi}{3}\right],\ \text{Sin}[60\,\text{Degree}],\ \text{Log}[E],\ \text{Exp}[1],\ \text{Exp}[\text{Log}[x]]\right\}$$

$$\left\{\frac{\sqrt{3}}{2},\ \frac{\sqrt{3}}{2},\ 1,\ e,\ x\right\}$$

```
{Abs[-6], Abs[0], Abs[6]}
```

{6, 0, 6}

```
{Sign[-2.5], Sign[2.5], Sign[0]}
```

{-1, 1, 0}

```
{Floor[-2.5], Floor[2.5], Floor[0]}
```

{-3, 2, 0}

▼ *Mathematica* can work as a calculator

★ Symbolic calculations and numerical calculations

When the number 1 is written, *Mathematica* understands that the number 1 is exact. It works automatically in the symbolic form

```
Exp[1]
```

ⅇ

When the number 1. is written, *Mathematica* does not consider 1.0 as the exact 1. *Mathematica* understands that it is the number 1 with its first 10 decimals equal to zero, and it works in the numerical form

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```
Exp[1.]
```

```
2.71828
```

```
ArcSin[1 / 2]
```

$$\frac{\pi}{6}$$

```
ArcSin[0.5]
```

```
0.523599
```

★ N[□], N[□,□] and //N

Mathematica works with 19 significative digits. If numerical data are introduced, *Mathematica* calculates the value with the accuracy that we have specified.

```
Sin[1 / 2.] // N
```

```
0.479426
```

```
N[Sin[1 / 2]]
```

```
0.479426
```

```
N[E, 40]
```

```
2.7182818284590452353602874713526624977572
```

★ Floating-point arithmetic

```
1.0000000000000123
```

```
1.0000000000000012
```

The following numbers are the same for *Mathematica:*

```
1.0000000000000123 // N
```

```
1.
```

```
1.0000000000000567 // N
```

```
1.
```

Local truncation error is done when the last digits are not considered. It has to be taken into account also that when *Mathematica* shows a number of 6 digits, a rounding error is done.

```
1.23400000000000789 // N
```

```
1.234
```

```
1.2345678000000789 // N
```

```
1.23457
```

The floating-point arithmetic means that depending on the size of the number the position of the point is changed

```
123.45678000000789 // N
```

```
123.457
```

```
123456.78000000789 // N
```

```
123457.
```

```
1234567800.0000789 // N
```

$$1.23457 \times 10^{9}$$

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▼ High-level programming language

★ *Mathematica* has subprograms for numerical calculations

```
NSolve[x^5+2==0,x]
```

```
{{x → -1.1487}, {x → -0.354967 - 1.09248 i}, {x → -0.354967 + 1.09248 i},
 {x → 0.929316 - 0.675188 i}, {x → 0.929316 + 0.675188 i}}
```

```
FindRoot[x^5 + 2 == 0, {x, 0.5}]
```

```
{x → -1.1487}
```

```
Integrate[Sin[x]^2,{x,0,Pi}]
```

$$\frac{\pi}{2}$$

```
NIntegrate[Sin[x]^2,{x,0,Pi}]
```

```
1.5708
```

```
NIntegrate[1/Log[x],{x,2,10}]
```

```
5.12044
```

★ Some elements to program in *Mathematica*

```
a=2;
If[a<1,2^2,b=Table[3+i,{i,1,3}]];
Print["b=",b]
```

```
b={4, 5, 6}
```

```
For[i = 0, i < 4, i++, Print[i]]
```

```
0
1
2
3
```

```
Do[Print[i], {i, 1, 9, 2}]
```

```
1
3
5
7
9
```

```
i = 0; While[i ≤ 3, i = i + 1; Print[i]]
```

```
1
2
3
4
```


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1.3. Functions and variables defined by the user

▼ Giving a value to a variable

The name of a varibale can be any alphanumeric chain

```
a=5;
```

```
b = a + 7;
```

```
?a
```

```
Global`a
```

```
a = 5
```

```
Clear[a]
```

```
?a
```

```
Global`a
```

```
? b
```

```
Global`b
```

```
b = 12
```

```
a + b /. a -> 7
```

```
19
```

```
sol = Solve[x^2 == 1]
```

```
{{x → -1}, {x → 1}}
```

```
sol[[1]]
```

```
{x → -1}
```

```
sol[[2,1]]
```

```
x → 1
```

```
erro1 = x /. sol[[1]]
```

```
-1
```

```
erro2 = sol[[2, 1, 2]]
```

```
1
```

▼ Function definition

★ The variable has to be specified

```
f[x_] = x^2
```

$$x^2$$

★ The name of a function can be any alphanumeric chain

```
fun1[x_] = (2 * x^3 - 1) / (Sqrt[3 * x] - a)
```

$$\frac{-1 + 2\,x^3}{-a + \sqrt{3}\,\sqrt{x}}$$

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★ Multiple variable functions

```
fun2[x_, a_] = (2 * x^3 - 1) / (√(3 * x) - a)
```

$$\frac{-1+2x^3}{-a+\sqrt{3}\ \sqrt{x}}$$

★ Asking for information about a function

```
? f
```

```
Global`f
```

```
f[x_] = x^2
```

★ Deleting a function

```
Clear[f]
```

```
? f
```

```
Global`f
```

```
Clear[$Line]
```

```
Clear["Global`*"]
```

★ Function evaluation

```
fun1[2]
```

$$\frac{15}{\sqrt{6}-a}$$

```
fun2[2, 1]
```

$$\frac{15}{-1+\sqrt{6}}$$

```
fun1[2] /. a → 1
```

$$\frac{15}{-1+\sqrt{6}}$$

▼ Piecewise functions

Piecewise functions can be defined using the commands “If”, “Which” or “Piecewise”

★ If [condition, value1, value2]

If the condition is true, the value “value1” is assigned, otherwise “value2”

```
abs[x_] = If[x < 0, -x, x]
```

```
If[x < 0, -x, x]
```

```
abs[2]
```

```
2
```

```
abs[-2]
```

```
2
```

```
abs /@ {-1, 0, 1}
```

```
{1, 0, 1}
```


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```
abs /@ Table[n, {n, -5, 5}]
```

```
{5, 4, 3, 2, 1, 0, 1, 2, 3, 4, 5}
```

★ Which [condition1, value1, condition2, value2, ..., conditionn, valuen]

It evaluates each of the coditions adequating the value that corresponds to the one that is true

```
g[x_] = Which[x < 0, x^2 - 4, x == 0, 5, x > 0, x + 3];
g /@ {-1, 0, 1, 3, 9}
```

```
{-3, 5, 4, 6, 12}
```

Or also in this way

```
g[x_] = Which[x ≤ 0, 1 - x^2, 0 < x < 1, x, True, x^2];
g /@ {-1, 0, 1, 3, 9}
```

```
{0, 1, 1, 9, 81}
```

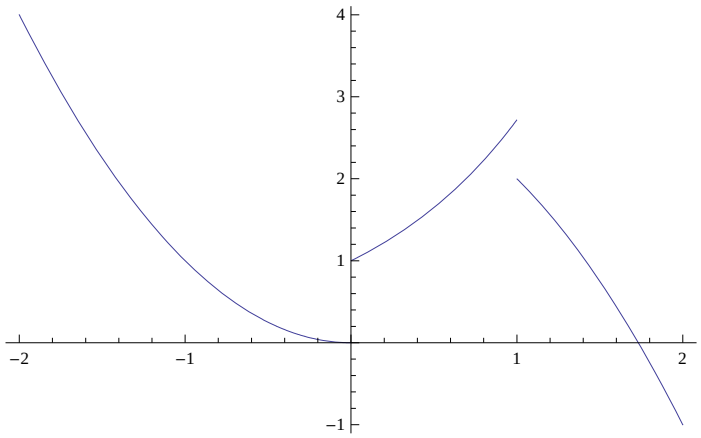
★ Piecewise [{{value$_1$, condition$_1$}, {value$_2$, condition$_2$}, ...}]

It represents each piece of the function in the domain specified in the condition

```
h[x_] = Piecewise[{{x^2, x < 0}, {E^x, 0 < x < 1}, {3 - x^2, x ≥ 1}}];
h /@ {-1, 0, 1, 3, 9}
```

```
{1, 0, 2, -6, -78}
```

```
Plot[h[x], {x, -2, 2}]
```



▼ Function operations

★ Algebraic operations

```
f[x_] = x^2; g[x_] = 2 * Sin[x];
{f[x] + g[x], f[x] * g[x], g[f[x]]}
```

```
{x^2 + 2 Sin[x], 2 x^2 Sin[x], 2 Sin[x^2]}
```

★ Limit calculations: Limit [function, {x,xmin,xmax}, x→x$_0$}

It gives the value to which the function tends when the variable x goes to the value x_0, or an interval (domain) of the values of the limit

```
Limit[x + 4, x → 2]
```

```
6
```

```
Limit[e^x, x → ∞]
```

```
∞
```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```
Limit[Sin[1/x], x → 0]
```

```
Interval[{-1, 1}]
```

We can use function with parameters

```
Limit[x^a/x^4, x → 0]
```

```
Limit[x^(-4+a), x → 0]
```

```
Limit[x^a/x^4, x → 0, Assumptions → a == 4]
```

```
1
```

```
Limit[x^a/x^4, x → 0, Assumptions → a > 4]
```

```
0
```

```
Limit[x^a/x^4, x → 0, Assumptions → a < 4]
```

```
∞
```

```
Limit[x^a/x^4, x → 0, Assumptions → 1 < a < 4]
```

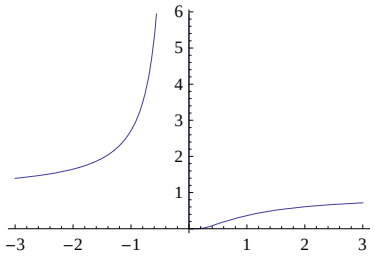
```
∞
```

Left-hand limit

```
f[x_] = E^(-1/x)
```

```
ℯ^(-1/x)
```

```
Plot[f[x], {x, -3, 3}]
```




```
Limit[f[x], x → 0, Direction → 1]
```

```
∞
```

Right-hand limit

```
Limit[f[x], x → 0, Direction → -1]
```

```
0
```

If we do not specify that we want to calculate the left- or the right-hand limit, the program calculates the right-hand limit by default

```
Limit[f[x], x → 0]
```

```
0
```

```
g[x_, y_] = x * y^2
```

```
x y^2
```

Repeated limits

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```
Limit[g[x, y], x → 1]
```

y^2

```
Limit[Limit[g[x, y], x → 1], y → 2]
```

4

★ Derivatives

The derivative of a one variable function

```
f[x_] = x^2;
```

```
D[f[x],x]
```

2 x

```
f'[x]
```

2 x

```
f''[x]
```

2

```
D[f[x], {x, 2}]
```

2

Partial derivatives

```
g[x_, y_] = x^2 * y^2;
```

```
D[g[x, y], x]
```

$2\,x\,y^2$

```
D[g[x, y], y]
```

$2\,x^2\,y$

```
D[g[x, y], {y, 2}]
```

$2\,x^2$

```
∂x,y g[x, y]
```

4 x y

★ Integration

Indefinite integrals

```
Integrate[Sin[x]^2, x]
```

$\frac{x}{2} - \frac{1}{4}\,\text{Sin}[2\,x]$

```
∫Sin[x]^2 ⅆx
```

$\frac{x}{2} - \frac{1}{4}\,\text{Sin}[2\,x]$

```
Integrate[f[x], x]
```

$\frac{x^3}{3}$

```
Integrate[g[x, y], x]
```

$\frac{x^3\,y^2}{3}$

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```
Integrate[Sin[x]^2, {x, 0, Pi}]
```

$$\frac{\pi}{2}$$

$$\int_0^{\pi} \text{Sin}[x]^2 \, dx$$

$$\frac{\pi}{2}$$

★ Resolution of algebraic equations

```
Solve[x^5 + 2 == 0, x]
```

$$\left\{\left\{x \to (-2)^{1/5}\right\}, \left\{x \to -2^{1/5}\right\}, \left\{x \to -(-1)^{2/5} 2^{1/5}\right\}, \left\{x \to (-1)^{3/5} 2^{1/5}\right\}, \left\{x \to -(-1)^{4/5} 2^{1/5}\right\}\right\}$$

```
Solve[1 - 2 * Sin[x] == 0, x]
```

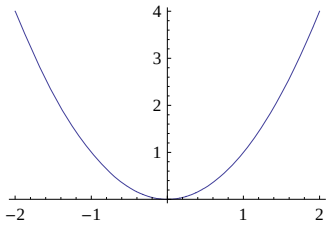
Solve::ifun: Inverse functions are being used by Solve, so
some solutions may not be found; use Reduce for complete solution information. >>

$$\left\{\left\{x \to \frac{\pi}{6}\right\}\right\}$$

★ Graphical representation of a function [function , {x, xmin, xmax}]

More than one explicit function can be plotted in the same axis

```
Plot[x^2, {x, -2, 2}]
```



★ Graphical representation of many functions [function , {x, xmin, xmax}] [{function1, function2,...., functionn} , {x, xmin, xmax}]

```
Plot[{x^2, x^3}, {x, -2, 2}]
```