P7

7. PRAKTIKA: ALDAGAI ANITZEKO FUNTZIOEN ADIERAZPEN GRAFIKOA

▼ Proposatutako Ariketa P- 7.1

Aztertu ondoko funtzioaren limitearen existentzia (0,0) puntuan:

$$f(x,y)=\frac{x\,y^2}{x^2+y^4}$$

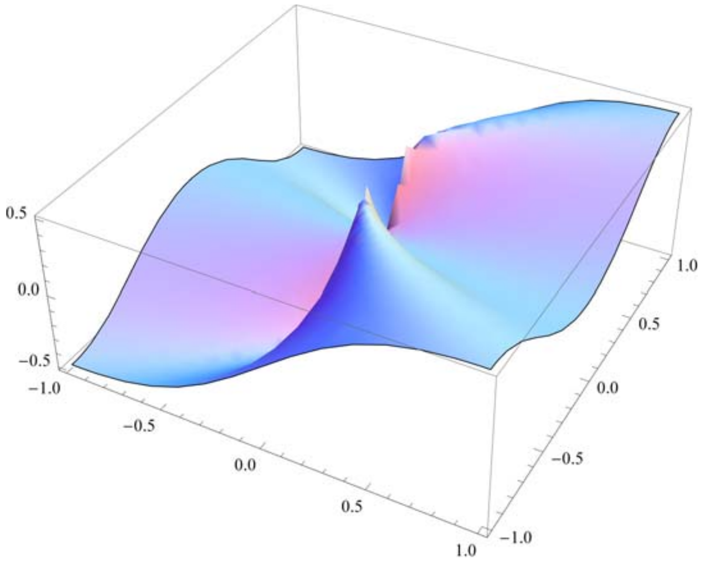
▼ Soluzioa P- 7.1

★ Funtzioa definituko dugu eta bere adierazpen grafikoa egingo dugu

```
Clear["Global`*"]
f[x_, y_] = (x * y^2) / (x^2 + y^4)
```

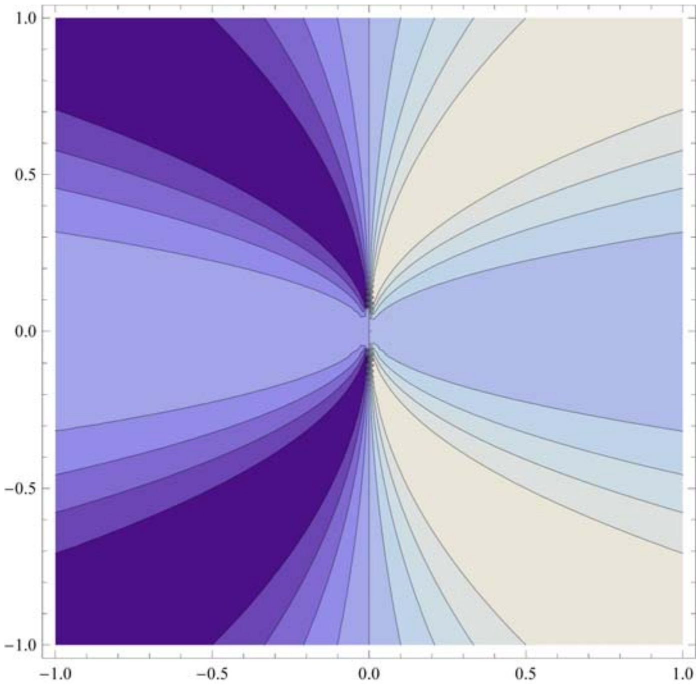
$$\frac{x\,y^2}{x^2+y^4}$$

```
Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1}, Mesh → False]
```



★ Maila-kurbak irudikatuko ditugu

```
ContourPlot[f[x, y], {x, -1, 1}, {y, -1, 1}]
```



Maila-kurbak parabolak direnez, ez da existituko limiterik

★ Limitearen kalkulua

<u>Errepikatutako limiteak</u>

```
(* Ez da existitzen f2[y] funtzio marjinala x->0-ra doanean *)

l1 = Limit[Limit[f[x, y], x -> 0], y -> 0]

0

l2 = Limit[Limit[f[x, y], y -> 0], x -> 0]

0
```

<u>Norabide limiteak</u>

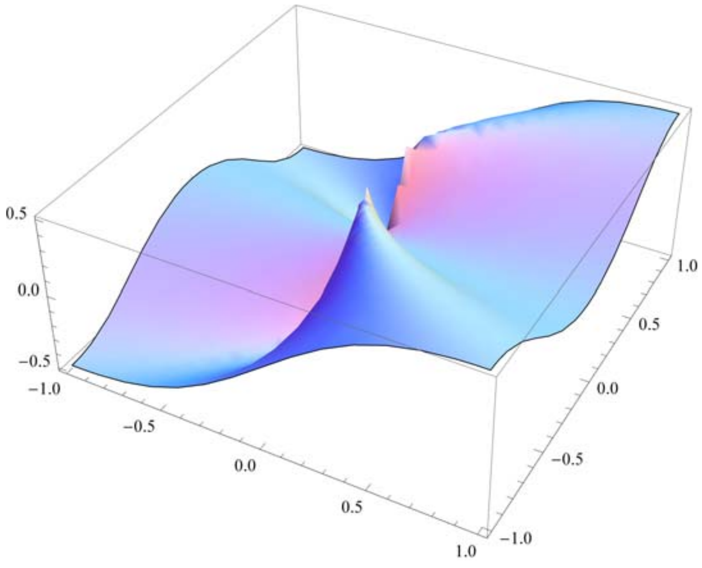
```
Limit[f[x, m * x], x -> 0]

0

(* Limite erradialak ez dira existitzen *)
```

<u>Paraboletan zeharreko norabide limiteak</u>

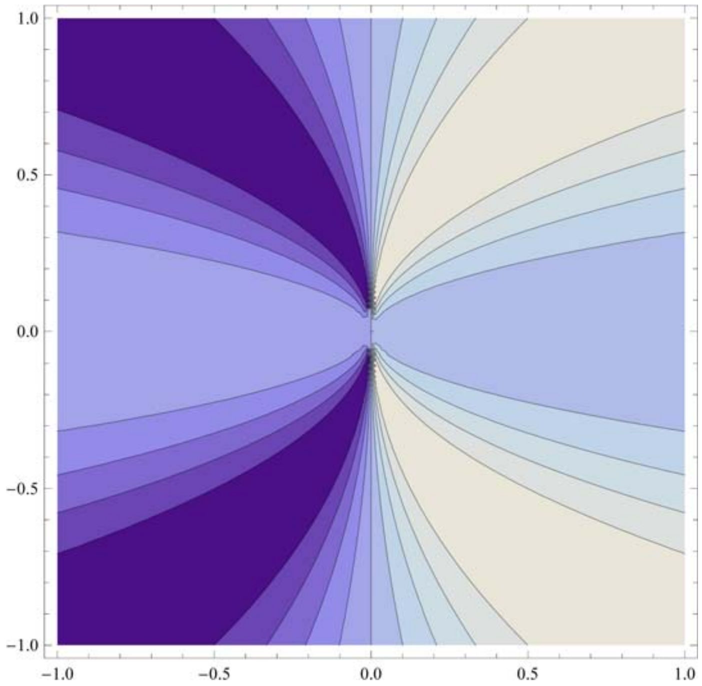
```
Limit[f[m * y^2, y], y -> 0]
```

$$\frac{m}{1 + m^2}$$

```
Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1}, Mesh → False]
```




```
ContourPlot[f[x, y], {x, -1, 1}, {y, -1, 1}]
```



▼ Proposatutako Ariketa P- 7.2

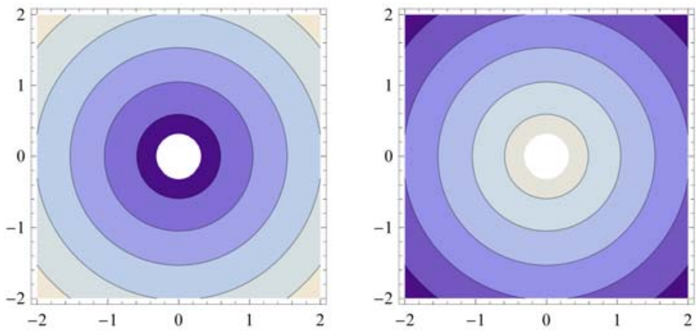
Egin ondoko hiperboloidearen adierazpen grafikoa $x^2 + y^2 - z^2 = 0.1$ eta irudikatu koordenatu ardatzekiko paraleloak diren plano paraleloekiko sekzioak.

▼ Soluzioa P- 7.2

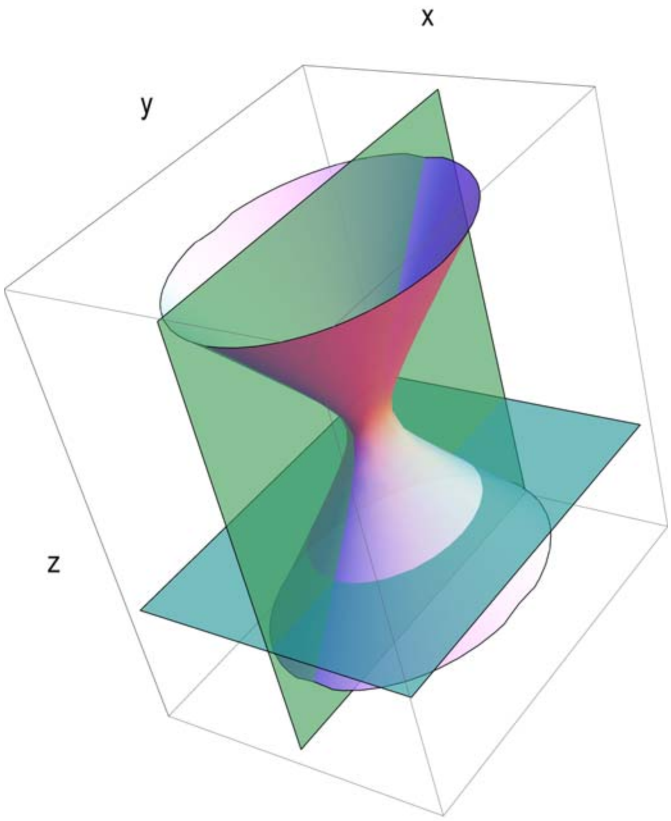
★ **Hiperboloidea definitzen duten bi aldagaitako funtzioak definituko ditugu eta beraien adierazpen grafikoa egingo dugu**

```
hip1 = ContourPlot[Sqrt[(x^2 + y^2 - 0.1)], {x, -2, 2}, {y, -2, 2}];

hip2 = ContourPlot[-Sqrt[(x^2 + y^2 - 0.1)], {x, -2, 2}, {y, -2, 2}];
```

```
Show[GraphicsGrid[{{hip1, hip2}}, Spacings → Scaled[0.2]]]
```




```
g2 = ContourPlot3D[{z == -1.2}, {x, -2.5, 2.5}, {y, -2.5, 2.5}, {z, -2.5, 2.5},
    ContourStyle → Directive[RGBColor[0.2, 0.8, 0.5], Opacity[0.6]], Mesh → False];
g3 = ContourPlot3D[{x == 0}, {x, -2.5, 2.5}, {y, -2.5, 2.5}, {z, -2.5, 2.5},
    ContourStyle → Directive[RGBColor[0.2, 0.8, 0.5], Opacity[0.6]], Mesh → False];
g4 = ContourPlot3D[x^2 + y^2 - 0.1 == z^2,
    {x, -2, 2}, {y, -2, 2}, {z, -2, 2}, Mesh → None, Ticks → None,
    ContourStyle → Directive[Opacity[0.8]], AxesLabel → {"X", "Z", "y"}];
g6 = Show[g2, g3, g4, Ticks → None, AxesLabel → {"X", "y", "Z"},
  BoxRatios → {1, 1.3, 1.5}, PlotRange → {-2.5, 2.5}]
```



▼ Proposatutako Ariketa P- 7.3

Aztertu ondorengo funtzioaren maximo eta minimoak grafikoki:

$$f(x,y)=x^3 + 3 * x * y^2 - 15 * x - 12 * y$$

▼ Soluzioa P- 7.3

★ Puntu geldikor edo estazionarioak kalkulatuko ditugu

Funtzioa definituko dugu

```
f[x_, y_] = x^3 + 3 * x * y^2 - 15 * x - 12 * y
```

$-15\,x + x^3 - 12\,y + 3\,x\,y^2$

Lehen ordenako deribatu partzialak ematen dizkiguten funtzioak definituko ditugu

```
dfx[x_, y_] = ∂x f[x, y]
```

$-15 + 3\,x^2 + 3\,y^2$

```
dfy[x_, y_] = ∂y f[x, y]
```

$-12 + 6\,x\,y$

```
gradf[x_, y_] = {dfx[x, y], dfy[x, y]}
```

$\{-15 + 3\,x^2 + 3\,y^2,\ -12 + 6\,x\,y\}$

Ekuazio sistema ebatziko dugu

$$\text{gradf}= 0 \Longleftrightarrow \begin{cases} f_x(x, y) = 0 \\ f_y(x, y) = 0 \end{cases}$$

```
s = Solve[gradf[x, y] == {0, 0}]
```

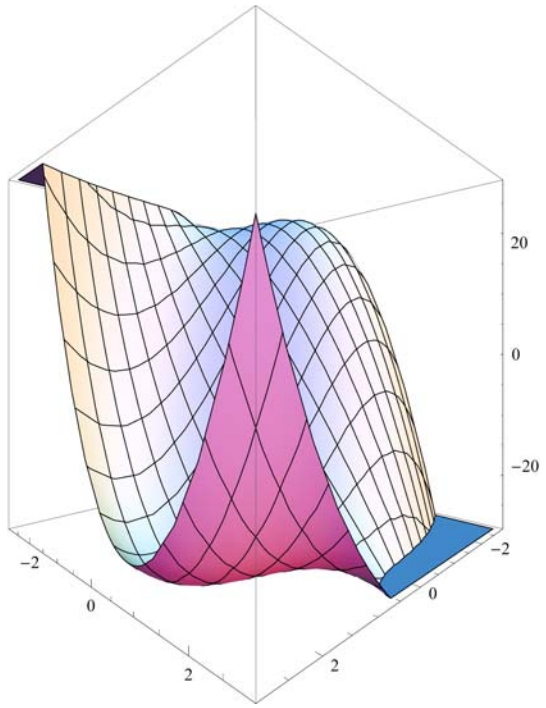
{{x → -2, y → -1}, {x → -1, y → -2}, {x → 1, y → 2}, {x → 2, y → 1}}

Ekuazio sistemaren emaitza diren puntuak definituko ditugu

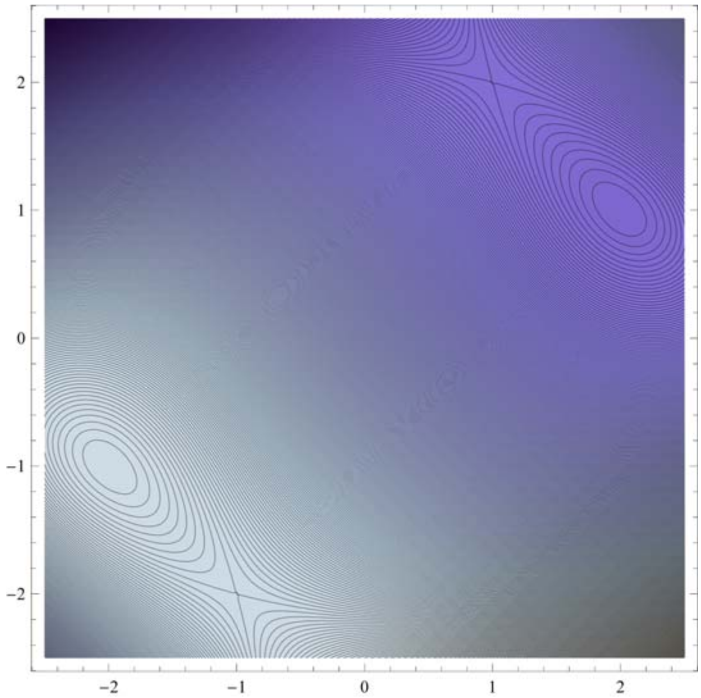
```
pc[n_] := {x, y} /. s[[n]];
```

⇒Puntu bakoitzaren inguruko funtzioaren eta maila-kurben adierazpen grafikoa egingo dugu

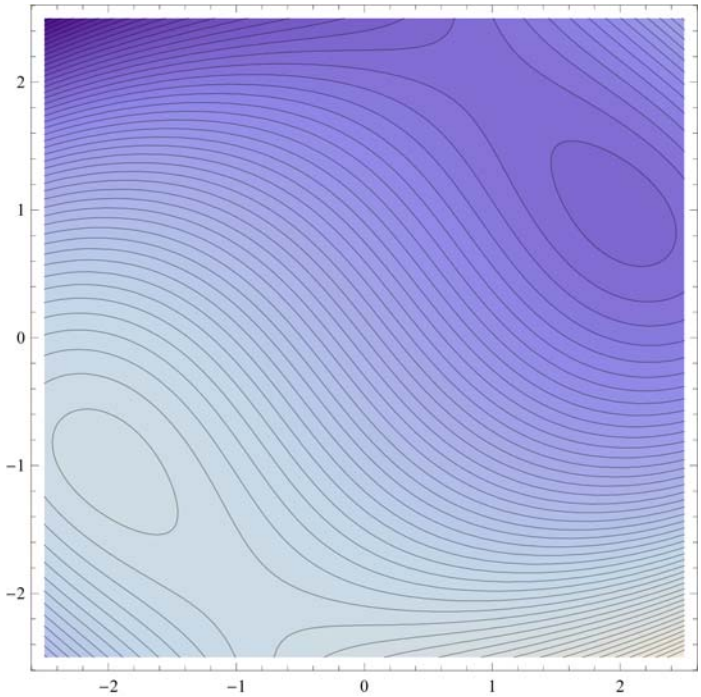
```
Plot3D[f[x, y], {x, -2.8, 2.8}, {y, -2.8, 2.8},
 PlotRange → {-29, 29}, BoxRatios → {1, 1, 1}, ViewPoint → {1, 1, 0}]
```



```
ContourPlot[f[x, y], {x, -2.5, 2.5}, {y, -2.5, 2.5},
 Contours → Function[{min, max}, Range[min, max, 0.2]]]
```

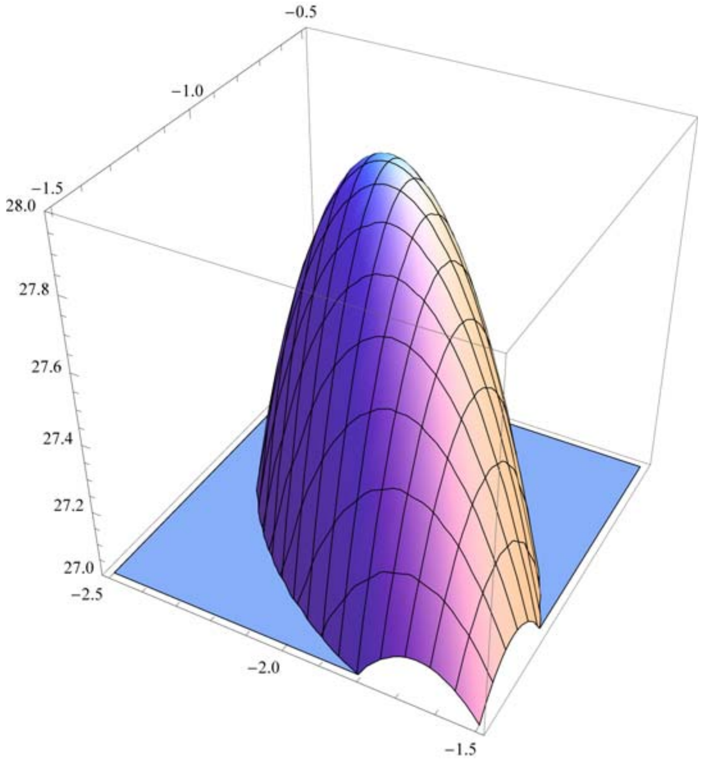



```
ContourPlot[f[x, y], {x, -2.5, 2.5}, {y, -2.5, 2.5}, Contours → 60]
```

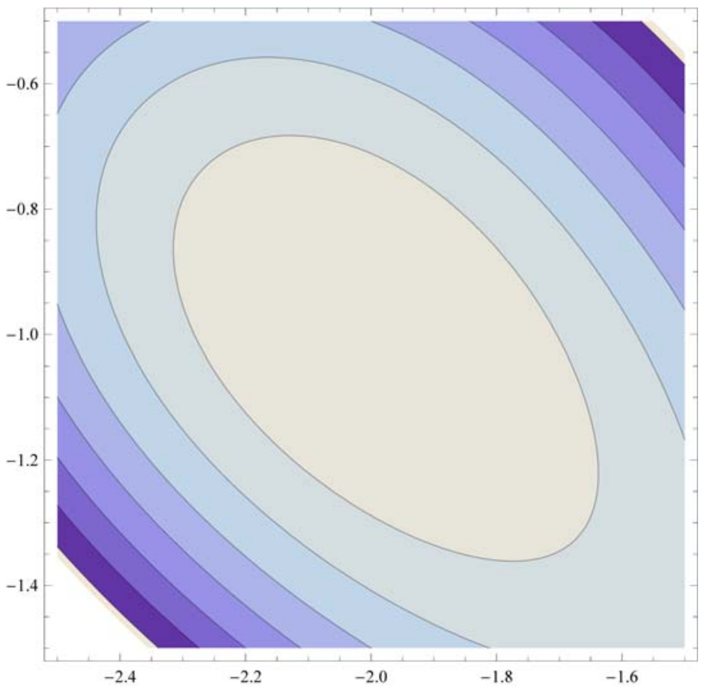


P[1] puntua maximo bat da

```
Plot3D[f[x, y], {x, -2.5, -1.5}, {y, -1.5, -0.5},
 PlotRange → {27, 28}, BoxRatios → {1, 1, 1}]
```

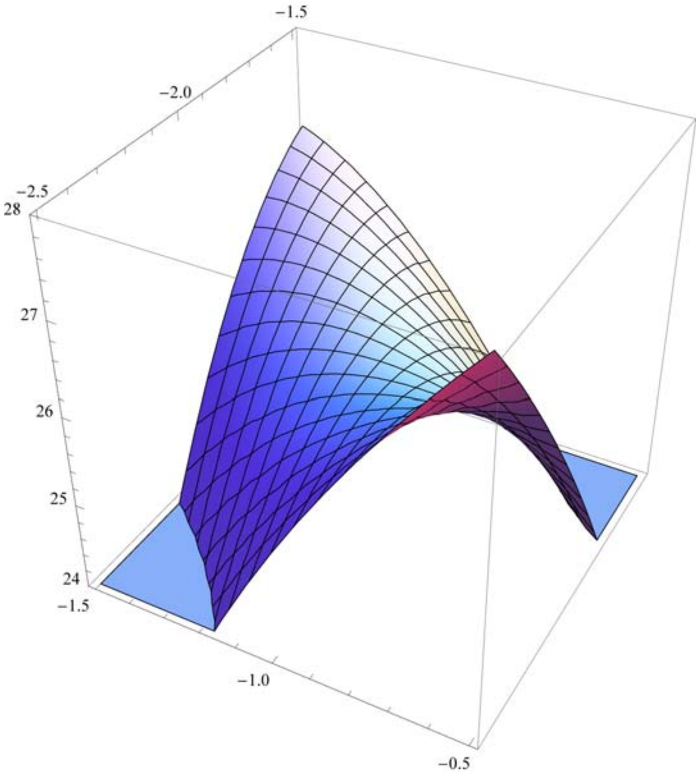



```
ContourPlot[f[x, y], {x, -2.5, -1.5}, {y, -1.5, -0.5}]
```

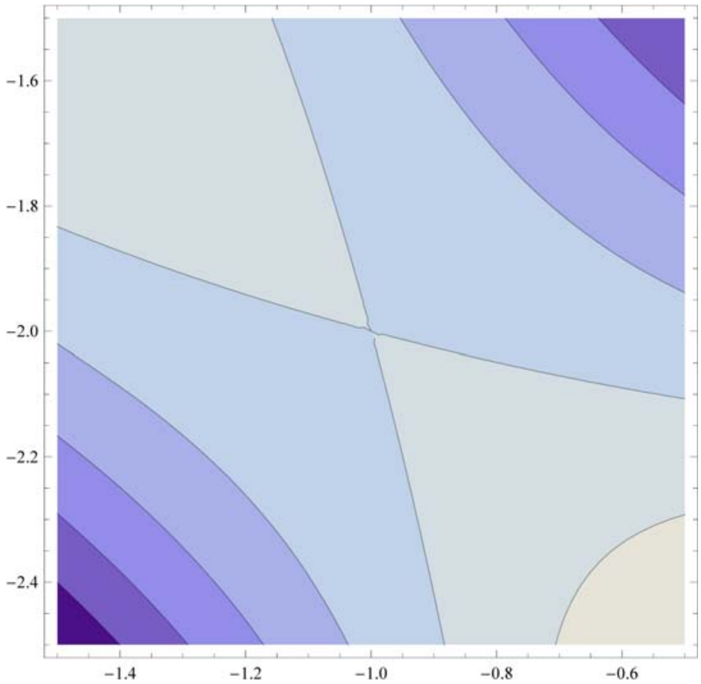


P[2] puntua zela-puntu bat da

```
Plot3D[f[x, y], {x, -1.5, -0.5}, {y, -2.5, -1.5},
 PlotRange → {24, 28}, BoxRatios → {1, 1, 1}]
```

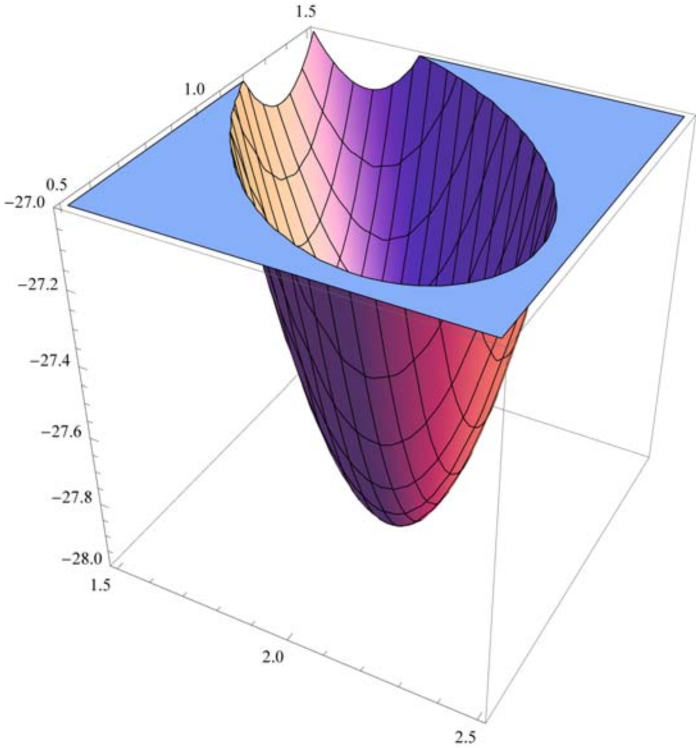



```
ContourPlot[f[x, y], {x, -1.5, -0.5}, {y, -2.5, -1.5}]
```

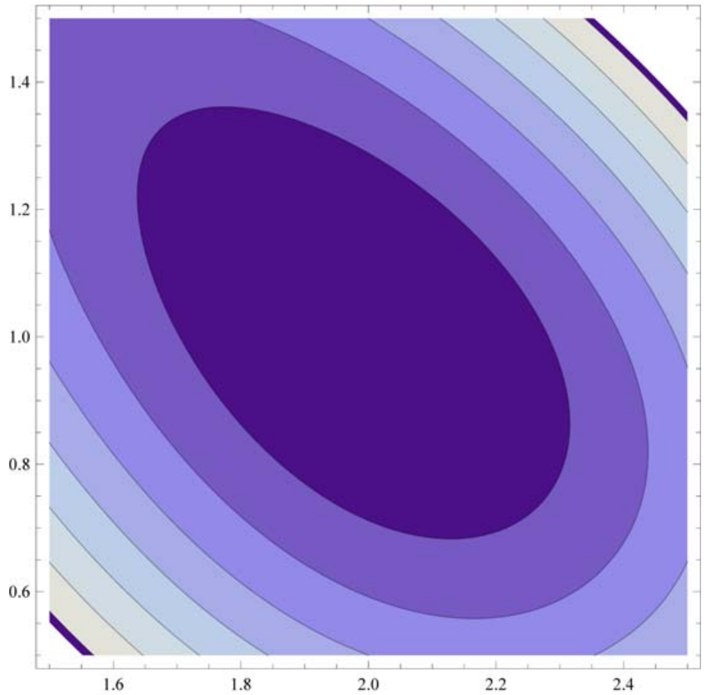


P[4] puntua minimo bat da

```
Plot3D[f[x, y], {x, 1.5, 2.5}, {y, 0.5, 1.5}, PlotRange → {-27, -28}, BoxRatios → {1, 1, 1}]
```




```
ContourPlot[f[x, y], {x, 1.5, 2.5}, {y, 0.5, 1.5}]
```



P[3] puntua zela-puntua da

```
f[1, 2]
```

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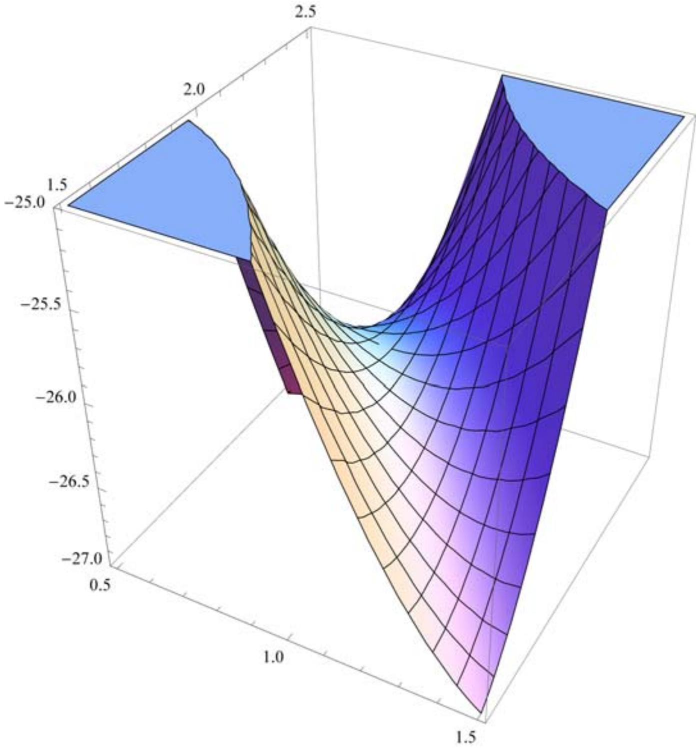
```
Plot3D[f[x, y], {x, 0.5, 1.5}, {y, 1.5, 2.5}, PlotRange → {-25, -27}, BoxRatios → {1, 1, 1}]
```




```
ContourPlot[f[x, y], {x, 0.5, 1.5}, {y, 1.5, 2.5}]
```