3. Gaia: Programen egiaztapena

3. Ariketa-orria: Asignazioak, konposaketa sekuentziala eta baldintzazkoak

1. Ondoko baieztapenetan post-baldintzak ({ ___ }) bete:

1.1.

$\{ \, true \, \}$
```
if x > y then
   z := x;
else
   z := y;
end if;
```
$\{ \, \underline{z = max(x, y)} \, \}$

1.2.

$\{ \, 1 \leq i \leq n \ \wedge \ z = \mathcal{N}k \ (\ 1 \leq k < i \ \wedge \ A(k) > B(k) \ ) \, \}$
```
if A(i) > B(i) then
   z := z+1;
end if;
```
$\{ \, \underline{z = \mathcal{N}k \ (\ 1 \leq k \leq i \ \wedge \ A(k) > B(k) \ )} \, \}$

2. Idatzi post-baldintza bete dadin exekutatu beharreko agindua (____;):

2.1.

$\{ \, i = n \ \wedge \ x \notin A(1..n-1) \, \}$
```
if  A(i) = x  then
    dago := true ;
else
    dago := false ;
end if;
```
$\{ \, dago \leftrightarrow x \in A(1..n) \, \}$

2.2.

$\{ \, 1 \leq i < n \ \wedge \ m = max(A(1..i)) \, \}$
```
i := i+1;
if  m < A(i)  then
    m := A(i) ;
end if;
```
$\{ \, 1 \leq i \leq n \ \wedge \ m = max(A(1..i)) \, \}$

3. Hurrengo frogapenetan hutsuneak bete (___):

3.1.

$\{ \, 1 \leq i < n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i) \, \}$
```
i := i+1;
if A(i) = x then
   dago := true;
end if;
```
$\{ \, 1 \leq i \leq n \ \wedge \ (\ dago \leftrightarrow x \in A(1..i) \ ) \, \}$

Frogapena:

1. $(1 \le i < n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i))$
   $\rightarrow (1 \le i+1-1 < n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i+1-1))$

2. $\{ 1 \le i+1-1 < n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i+1-1) \}$
   ```
   i := i+1;
   ```
   $\{ \underline{1 \le i-1 < n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1)} \}$ **(AA)**

3. $(1 \le i-1 < n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1))$
   $\rightarrow (1 \le i \le n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1))$

4. $\{ 1 \le i < n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i) \}$
   ```
   i := i+1;
   ```
   $\{ \underline{1 \le i \le n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1)} \}$ 1, 2, 3 eta **(ODE)**

5. $(1 \le i \le n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1) \ \wedge \ A(i) = x)$
   $\rightarrow (1 \le i \le n \ \wedge \ x \in A(1..i))$
   $\rightarrow (1 \le i \le n \ \wedge \ (true \leftrightarrow x \in A(1..i)))$

6. $\{ \underline{1 \le i \le n \ \wedge \ (true \leftrightarrow x \in A(1..i))} \}$
   ```
   dago := true;
   ```
   $\{ \underline{1 \le i \le n \ \wedge \ (dago \leftrightarrow x \in A(1..i))} \}$ **(AA)**

7. $\{ 1 \le i \le n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1) \ \wedge \ A(i) = x \}$
   ```
   dago := true;
   ```
   $\{ 1 \le i \le n \ \wedge \ (dago \leftrightarrow x \in A(1..i)) \}$ 5, 6 eta **(ODE)**

8. $(\underline{1 \le i \le n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1) \ \wedge \ A(i) \ne x})$
   $\rightarrow (1 \le i \le n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i))$
   $\rightarrow (1 \le i \le n \ \wedge \ (dago \leftrightarrow x \in A(1..i)))$

9. $(1 \le i \le n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1)) \rightarrow (\underline{1 \le i \le n})$
   $\rightarrow def(A(i) = x)$

10. $\{ 1 \le i \le n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i-1) \}$
    ```
    if A(i) = x then
       dago := true;
    end if;
    ```
    $\{ 1 \le i \le n \ \wedge \ (dago \leftrightarrow x \in A(1..i)) \}$ 7, 8, 9 eta **(BDE)**

11. $\{ 1 \le i < n \ \wedge \ \neg dago \ \wedge \ x \notin A(1..i) \}$
    ```
    i := i+1;
    if A(i) = x then
       dago := true;
    end if;
    ```
    $\{ 1 \le i \le n \ \wedge \ (dago \leftrightarrow x \in A(1..i)) \}$ 4, 10 eta **(KPE)**

3.2. $\{\ 1 \leq err < n\ \wedge\ 1 \leq zut \leq n\ \wedge\ (err-1) \times n + zut = b\ \}$

```
if zut = n then
   err := err+1;
   zut := 1;
else
   zut := zut+1;
end if;
```

$\{\ 1 \leq err, zut \leq n\ \wedge\ (err-1) \times n + zut = b+1\ \}$

Frogapena:

1. $(\ 1 \leq err < n\ \wedge\ 1 \leq zut \leq n\ \wedge\ (err-1) \times n + zut = b\ \wedge\ zut = n\ )$
   $\rightarrow\ (\ 1 \leq err < n\ \wedge\ (err-1) \times n + n = b\ )$
   $\rightarrow\ (\ 1 \leq err < n\ \wedge\ err \times n = b\ )$
   $\rightarrow\ (\ 1 \leq err+1-1 < n\ \wedge\ (err+1-1) \times n = b\ )$

2. $\{\ \underline{1 \leq err+1-1 < n\ \wedge\ (err+1-1) \times n = b}\ \}$
   `err := err+1;`
   $\{\ 1 \leq err-1 < n\ \wedge\ (err-1) \times n = b\ \}$ **(AA)**

3. $(\ 1 \leq err-1 < n\ \wedge\ (err-1) \times n = b\ )$
   $\rightarrow\ (\ 1 \leq err \leq n\ \wedge\ (err-1) \times n = b\ )$

4. $\{\ 1 \leq err < n\ \wedge\ 1 \leq zut \leq n\ \wedge\ (err-1) \times n + zut = b\ \wedge\ zut = n\ \}$
   `err := err+1;`
   $\{\ 1 \leq err \leq n\ \wedge\ (err-1) \times n = b\ \}$ 1, 2, 3 eta **(ODE)**

5. $\{\ 1 \leq err \leq n\ \wedge\ (err-1) \times n = b\ \}$
   `zut := 1;`
   $\{\ \underline{1 \leq err \leq n\ \wedge\ (err-1) \times n = b\ \wedge\ zut = 1}\ \}$ **(AA)**

6. $(\ 1 \leq err \leq n\ \wedge\ (err-1) \times n = b\ \wedge\ zut = 1\ )$
   $\rightarrow\ (\ 1 \leq err \leq n\ \wedge\ (err-1) \times n + 1 = b+1\ \wedge\ zut = 1\ )$
   $\rightarrow\ (\ 1 \leq err, zut \leq n\ \wedge\ (err-1) \times n + zut = b+1\ )$

7. $\{\ 1 \leq err \leq n\ \wedge\ (err-1) \times n = b\ \}$
   `zut := 1;`
   $\{\ 1 \leq err, zut \leq n\ \wedge\ (err-1) \times n + zut = b+1\ \}$ 5, 6 eta **(ODE)**

8. $\{\ 1 \leq err < n\ \wedge\ 1 \leq zut \leq n\ \wedge\ (err-1) \times n + zut = b\ \wedge\ zut = n\ \}$
   `err := err+1;`
   `zut := 1;`
   $\{\ 1 \leq err, zut \leq n\ \wedge\ (err-1) \times n + zut = b+1\ \}$ <u>4, 7 eta **(KPE)**</u>

9. $(\ \underline{1 \leq err < n\ \wedge\ 1 \leq zut \leq n\ \wedge\ (err-1) \times n + zut = b\ \wedge\ zut \neq n}\ )$
   $\rightarrow\ (\ 1 \leq err < n\ \wedge\ 1 \leq zut < n\ \wedge\ (err-1) \times n + zut = b\ )$
   $\rightarrow\ (\ 1 \leq err < n\ \wedge\ 1 \leq zut+1-1 < n\ \wedge$
   $(err-1) \times n + zut + 1 - 1 = b\ )$

10. $\{\ \underline{1 \leq err < n \wedge 1 \leq zut+1-1 < n \wedge (err-1) \times n + zut + 1 - 1 = b}\ \}$
    `zut := zut+1;`
    $\{\ 1 \leq err < n\ \wedge\ 1 \leq zut-1 < n\ \wedge\ (err-1) \times n + zut - 1 = b\ \}$ **(AA)**

11. $(\ 1 \leq err < n \ \wedge \ 1 \leq zut - 1 < n \ \wedge \ (err - 1) \times n + zut - 1 = b \ )$
$\rightarrow \ (\ 1 \leq err < n \ \wedge \ 1 \leq zut \leq n \ \wedge \ (err - 1) \times n + zut = b + 1 \ )$
$\rightarrow \ (\ 1 \leq err, zut \leq n \ \wedge \ (err - 1) \times n + zut = b + 1 \ )$

12. $\{ \ 1 \leq err < n \ \wedge \ 1 \leq zut \leq n \ \wedge \ (err - 1) \times n + zut = b \ \wedge \ zut \neq n \ \}$

```
zut := zut+1;
```

$\{ \ 1 \leq err, zut \leq n \ \wedge \ (err - 1) \times n + zut = b + 1 \ \}$ — 9, 10, 11 eta **(ODE)**

13. $(\ 1 \leq err < n \ \wedge \ 1 \leq zut \leq n \ \wedge \ (err - 1) \times n + zut = b \ ) \ \rightarrow$
$def(zut = n)$

14. $\{ \ 1 \leq err < n \ \wedge \ 1 \leq zut \leq n \ \wedge \ (err - 1) \times n + zut = b \ \}$

```
if zut = n then
   err := err+1;
   zut := 1;
else
   zut := zut+1;
end if;
```

$\{ \ 1 \leq err, zut \leq n \ \wedge \ (err - 1) \times n + zut = b + 1 \ \}$ — 8, 12, 13 eta **(BDE)**

4. Egiaztatu ondoko baieztapenak:

4.1. $\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i - 1)) \ \}$

```
if m < A(i) then
   m := A(i);
end if;
i := i+1;
```

$\{ \ 1 < i \leq n + 1 \ \wedge \ m = max(A(1..i - 1)) \ \}$

Soluzioa:

1. $(\ 1 < i \leq n \ \wedge \ m = max(A(1..i - 1)) \ \wedge \ m < A(i) \ )$
$\rightarrow \ (\ 1 < i \leq n \ \wedge \ A(i) = max(A(1..i)) \ )$

2. $\{ \ 1 < i \leq n \ \wedge \ A(i) = max(A(1..i)) \ \}$

```
m := A(i);
```

$\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i)) \ \}$ — **(AA)**

3. $\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i - 1)) \ \wedge \ m < A(i) \ \}$

```
m := A(i);
```

$\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i)) \ \}$ — 1, 2 eta **(ODE)**

4. $(\ 1 < i \leq n \ \wedge \ m = max(A(1..i - 1)) \ \wedge \ m \geq A(i) \ )$
$\rightarrow \ (\ 1 < i \leq n \ \wedge \ m = max(A(1..i)) \ ) \ )$

5. $(\ 1 < i \leq n \ \wedge \ m = max(A(1..i - 1)) \ ) \ \rightarrow \ (\ 1 < i \leq n \ )$
$\rightarrow \ def(m < A(i))$

6. $\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i - 1)) \ \}$

```
if m < A(i) then
   m := A(i);
end if;
```

$\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i)) \ \}$ — 3, 4, 5 eta **(BDE)**

7. $\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i)) \ \}$

```
i := i+1;
```

$\{ \ 1 < i-1 \leq n \ \wedge \ m = max(A(1..i-1)) \ \}$ **(AA)**

8. $(\ 1 < i-1 \leq n \ \wedge \ m = max(A(1..i-1)) \ ) \ \rightarrow$
$(\ 1 < i \leq n+1 \ \wedge \ m = max(A(1..i-1)) \ )$

9. $\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i)) \ \}$

```
i := i+1;
```

$\{ \ 1 < i \leq n+1 \ \wedge \ m = max(A(1..i-1)) \ \}$ 7, 8 eta **(ODE)**

10. $\{ \ 1 < i \leq n \ \wedge \ m = max(A(1..i-1)) \ \}$

```
if m < A(i) then
   m := A(i);
end if;
i := i+1;
```

$\{ \ 1 < i \leq n+1 \ \wedge \ m = max(A(1..i-1)) \ \}$ 6, 9 eta **(KPE)**

4.2. $\{ \ true \ \}$

```
if x mod 2 = 0 then
   bikoiti := true;
else
   bikoiti := false;
end if;
```

$\{ \ bikoiti \leftrightarrow x \ mod \ 2 = 0 \ \}$

Soluzioa:

1. $\{ \ true \ \wedge \ x \ mod \ 2 = 0 \ \}$

```
bikoiti := true;
```

$\{ \ true \ \wedge \ x \ mod \ 2 = 0 \ \wedge \ bikoiti \ \}$ **(AA)**

2. $(\ true \ \wedge \ x \ mod \ 2 = 0 \ \wedge \ bikoiti \ ) \ \rightarrow \ (\ bikoiti \leftrightarrow x \ mod \ 2 = 0 \ )$

3. $\{ \ true \ \wedge \ x \ mod \ 2 = 0 \ \}$

```
bikoiti := true;
```

$\{ \ bikoiti \leftrightarrow x \ mod \ 2 = 0 \ \}$ 1, 2 eta **(ODE)**

4. $\{ \ true \ \wedge \ \neg(\ x \ mod \ 2 = 0 \ ) \ \}$

```
bikoiti := false;
```

$\{ \ true \ \wedge \ \neg(\ x \ mod \ 2 = 0 \ ) \ \wedge \ \neg bikoiti \ \}$ **(AA)**

5. $(\ true \ \wedge \ \neg(\ x \ mod \ 2 = 0 \ ) \ \wedge \ \neg bikoiti \ ) \ \rightarrow$
$(\ bikoiti \leftrightarrow x \ mod \ 2 = 0 \ )$

6. $\{ \ true \ \wedge \ \neg(\ x \ mod \ 2 = 0 \ \}$

```
bikoiti := false;
```

$\{ \ bikoiti \leftrightarrow x \ mod \ 2 = 0 \ \}$ 4, 5 eta **(ODE)**

7. $(\ true \ ) \ \rightarrow \ def(x \ mod \ 2 = 0)$

8. $\{ \, true \, \}$

```
if x mod 2 = 0 then
   bikoiti := true;
else
   bikoiti := false;
end if;
```

$\{ \, bikoiti \leftrightarrow x \; mod \; 2 = 0 \, \}$ 3, 6, 7 eta **(ODE)**

5. Frogatu zuzentasunari buruzko ondorengo baieztapenak, kontuan hartuz FIB_i-k Fibonacciren segidaren i-garren terminoa adierazten duela:

5.1. $\{ \, 1 \le i < n \; \wedge \; x = FIB_i \; \wedge \; y = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \, \}$

```
x := y;
y := z;
z := x+y;
i := i+1;
```

$\{ \, 1 \le i \le n \; \wedge \; x = FIB_i \; \wedge \; y = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \, \}$

Soluzioa:

1. $(\, 1 \le i < n \; \wedge \; x = FIB_i \; \wedge \; y = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \,)$
   $\rightarrow \; (\, 1 \le i < n \; \wedge \; y = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \,)$

2. $\{ \, 1 \le i < n \; \wedge \; y = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \, \}$
   `x := y;`
   $\{ \, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \, \}$ **(AA)**

3. $\{ \, 1 \le i < n \; \wedge \; x = FIB_i \; \wedge \; y = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \, \}$
   `x := y;`
   $\{ \, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \, \}$ 1, 2 eta **(ODE)**

4. $\{ \, 1 \le i < n \; \wedge \; y = FIB_{i+1} \; \wedge \; z = FIB_{i+2} \, \}$
   `y := z;`
   $\{ \, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \, \}$ **(AA)**

5. $(\, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \,)$
   $\rightarrow \; (\, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \; \wedge$
   $x + y = FIB_{i+1} + FIB_{i+2} \,)$
   $\rightarrow \; (\, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \; \wedge \; x + y = FIB_{i+3} \,)$

6. $\{ \, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \; \wedge \; x + y = FIB_{i+3} \, \}$
   `z := x+y;`
   $\{ \, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \; \wedge \; z = FIB_{i+3} \, \}$ **(AA)**

7. $\{ \, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \, \}$
   `z := x+y;` 5, 6 eta
   $\{ \, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \; \wedge \; z = FIB_{i+3} \, \}$ **(ODE)**

8. $(\, 1 \le i < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+2} \; \wedge \; z = FIB_{i+3} \,)$
   $\rightarrow \; (\, 1 \le i + 1 - 1 < n \; \wedge \; x = FIB_{i+1} \; \wedge \; y = FIB_{i+1+1} \; \wedge$
   $z = FIB_{i+1+2} \,)$

9. $\{ \ 1 \leq i+1-1 < n \ \wedge \ x = FIB_{i+1} \ \wedge \ y = FIB_{i+1+1} \ \wedge \ z = FIB_{i+1+2} \ \}$
```
i := i+1;
```
$\{ \ 1 \leq i-1 < n \ \wedge \ x = FIB_i \ \wedge \ y = FIB_{i+1} \ \wedge \ z = FIB_{i+2} \ \}$ **(AA)**

10. $(\ 1 \leq i-1 < n \ \wedge \ x = FIB_i \ \wedge \ y = FIB_{i+1} \ \wedge \ z = FIB_{i+2} \ )$
$\rightarrow \ (\ 1 \leq i \leq n \ \wedge \ x = FIB_i \ \wedge \ y = FIB_{i+1} \ \wedge \ z = FIB_{i+2} \ )$

11. $\{ \ 1 \leq i < n \ \wedge \ x = FIB_{i+1} \ \wedge \ y = FIB_{i+2} \ \wedge \ z = FIB_{i+3} \ \}$
```
i := i+1;
```
8, 9, 10 eta
$\{ \ 1 \leq i \leq n \ \wedge \ x = FIB_i \ \wedge \ y = FIB_{i+1} \ \wedge \ z = FIB_{i+2} \ \}$ **(ODE)**

12. $\{ \ 1 \leq i < n \ \wedge \ x = FIB_i \ \wedge \ y = FIB_{i+1} \ \wedge \ z = FIB_{i+2} \ \}$
```
x := y;
y := z;
z := x+y;
i := i+1;
```
3, 4, 7, 11 eta
$\{ \ 1 \leq i \leq n \ \wedge \ x = FIB_i \ \wedge \ y = FIB_{i+1} \ \wedge \ z = FIB_{i+2} \ \}$ **(KPE)**

5.2. $\{ \ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_i \ \wedge \ z = FIB_{i+1} \ ) \ \}$
```
u := u+z;
z := u+z;
```
$\{ \ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_i \ \wedge \ z = FIB_{i+1} \ ) \ \}$

Soluzioa:

1. $(\ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_i \ \wedge \ z = FIB_{i+1} \ ) \ )$
$\rightarrow \ (\ \exists i \ (\ i \geq 0 \ \wedge \ u+z-z = FIB_i \ \wedge \ z = FIB_{i+1} \ ) \ )$

2. $\{ \ \exists i \ (\ i \geq 0 \ \wedge \ u+z-z = FIB_i \ \wedge \ z = FIB_{i+1} \ ) \ \}$
```
u := u+z;
```
$\{ \ \exists i \ (\ i \geq 0 \ \wedge \ u-z = FIB_i \ \wedge \ z = FIB_{i+1} \ ) \ \}$ **(AA)**

3. $(\ \exists i \ (\ i \geq 0 \ \wedge \ u-z = FIB_i \ \wedge \ z = FIB_{i+1} \ ) \ )$
$\rightarrow \ (\ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_i + z \ \wedge \ z = FIB_{i+1} \ ) \ )$
$\rightarrow \ (\ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_i + FIB_{i+1} \ \wedge \ z = FIB_{i+1} \ ) \ )$
$\rightarrow \ (\ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_{i+2} \ \wedge \ z = FIB_{i+1} \ ) \ )$

4. $\{ \ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_i \ \wedge \ z = FIB_{i+1} \ ) \ \}$
```
u := u+z;
```
$\{ \ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_{i+2} \ \wedge \ z = FIB_{i+1} \ ) \ \}$ 1, 2, 3 eta **(ODE)**

5. $(\ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_{i+2} \ \wedge \ z = FIB_{i+1} \ ) \ )$
$\rightarrow \ (\ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_{i+2} \ \wedge \ u+z-u = FIB_{i+1} \ ) \ )$

6. $\{ \ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_{i+2} \ \wedge \ u+z-u = FIB_{i+1} \ ) \ \}$
```
z := u+z;
```
$\{ \ \exists i \ (\ i \geq 0 \ \wedge \ u = FIB_{i+2} \ \wedge \ z-u = FIB_{i+1} \ ) \ \}$ **(AA)**

7. $(\exists i\ (i \geq 0 \ \wedge\ u = FIB_{i+2} \ \wedge\ z - u = FIB_{i+1}))$
$\rightarrow (\exists i\ (i \geq 0 \ \wedge\ u = FIB_{i+2} \ \wedge\ z = FIB_{i+1} + u))$
$\rightarrow (\exists i\ (i \geq 0 \ \wedge\ u = FIB_{i+2} \ \wedge\ z = FIB_{i+1} + FIB_{i+2}))$
$\rightarrow (\exists i\ (i \geq 0 \ \wedge\ u = FIB_{i+2} \ \wedge\ z = FIB_{i+3}))$
$\rightarrow (\exists i\ (i \geq 2 \ \wedge\ u = FIB_{i} \ \wedge\ z = FIB_{i+1}))$
$\rightarrow (\exists i\ (i \geq 0 \ \wedge\ u = FIB_{i} \ \wedge\ z = FIB_{i+1}))$

8. $\{ \exists i\ (i \geq 0 \ \wedge\ u = FIB_{i+2} \ \wedge\ z = FIB_{i+1}) \}$
```
z := u+z;
```
$\{ \exists i\ (i \geq 0 \ \wedge\ u = FIB_{i} \ \wedge\ z = FIB_{i+1}) \}$ 5, 6, 7 eta **(ODE)**

9. $\{ \exists i\ (i \geq 0 \ \wedge\ u = FIB_{i} \ \wedge\ z = FIB_{i+1}) \}$
```
u := u+z;
z := u+z;
```
$\{ \exists i\ (i \geq 0 \ \wedge\ u = FIB_{i} \ \wedge\ z = FIB_{i+1}) \}$ 4, 8 eta **(KPE)**

6. Izan bedi ondoko Hoare-ren hirukotea:

$\{ \ true \ \}$
```
if x = 0 then
   y := 3;
end if;
```
$\{ \varphi \}$

Ondokoetatik zein da post-baldintza egokia?

$\{ x = 0 \ \wedge\ y = 3 \}$

$\{ x = 0 \ \leftrightarrow\ y = 3 \}$

$\{ x = 0 \ \rightarrow\ y = 3 \}$

$\{ y = 3 \ \rightarrow\ x = 0 \}$

Soluzioa:

$\{ \ true \ \}$
```
if x = 0 then
   y := 3;
end if;
```
$\{ x = 0 \ \wedge\ y = 3 \}$

$\{ \ true \ \}$
```
if x = 0 then
   y := 3;
end if;
```
$\{ x = 0 \ \leftrightarrow\ y = 3 \}$

$\{ \ true \ \}$
```
if x = 0 then
   y := 3;
end if;
```
$\{ y = 3 \ \rightarrow\ x = 0 \}$

baieztapenak ez dira zuzenak, $\{ x = 1 \ \wedge\ y = 3 \}$ formula betetzen den egoera batean hasten bagara, $\{ x = 1 \ \wedge\ y = 3 \}$ betetzen den egoera batean bukatuko garelako, eta:

$$(x = 1 \ \wedge\ y = 3) \ \not\rightarrow\ (x = 0 \ \wedge\ y = 3)$$
$$(x = 1 \ \wedge\ y = 3) \ \not\rightarrow\ (x = 0 \ \leftrightarrow\ y = 3)$$
$$(x = 1 \ \wedge\ y = 3) \ \not\rightarrow\ (y = 3 \ \rightarrow\ x = 0)$$

Hurrengo baieztapena, berriz, frogagarria da:

$\{ \, true \, \}$
```
   if x = 0 then
      y := 3;
   end if;
```
$\{ \, x = 0 \;\rightarrow\; y = 3 \, \}$

Frogapena:

1. $\{ \, true \;\wedge\; x = 0 \, \} \equiv \{ \, x = 0 \, \}$
   ```
   y := 3;
   ```
   $\{ \, x = 0 \;\wedge\; y = 3 \, \}$ **(AA)**

2. $(\, x = 0 \;\wedge\; y = 3 \,) \;\rightarrow\; (\, y = 3 \,) \;\rightarrow\; (\, x \neq 0 \;\vee\; y = 3 \,) \;\rightarrow\; (\, x = 0 \;\rightarrow\; y = 3 \,)$

3. $\{ \, true \;\wedge\; x = 0 \, \} \equiv \{ \, x = 0 \, \}$
   ```
   y := 3;
   ```
   $\{ \, x = 0 \;\rightarrow\; y = 3 \, \}$ 1, 2 eta **(ODE)**

4. $(\, true \;\wedge\; x \neq 0 \,) \;\rightarrow\; (\, x \neq 0 \,) \;\rightarrow\; (\, x \neq 0 \;\vee\; y = 3 \,) \;\rightarrow\; (\, x = 0 \;\rightarrow\; y = 3 \,)$

5. $(\, true \,) \;\rightarrow\; def(x = 0)$

6. $\{ \, true \, \}$
   ```
   if x = 0 then
      y := 3;
   end if;
   ```
   $\{ \, x = 0 \;\wedge\; y = 3 \, \}$ 3, 4, 5 eta **(BDE)**

7. Zuzena al da ondoko baieztapena?

$\{ \, true \, \}$
```
   if x < y then
      lag := x; x := y; y := lag;
   elsif y < z then
      lag := y; y := z; z := lag;
   else
      lag := x; x := z; z := lag;
   end if;
```
$\{ \, x \geq y \geq z \, \}$

Soluzioa: Ez da zuzena, $\{ \, x < y \;\wedge\; y < z \, \}$ formula betetzen den egoera batean hasten bagara, $\{ \, x > y \;\wedge\; y < z \, \}$ betetzen den egoera batean bukatuko garelako, eta

$$(\, x > y \;\wedge\; y < z \,) \;\not\rightarrow\; (\, x \geq y \geq z \,)$$

Eta beste hau?

$\{ \; true \; \}$

```
   if x < y then
      lag := x; x := y; y := lag;
   end if;
   if y < z then
      lag := y; y := z; z := lag;
   end if;
   if x < y then
      lag := x; x := y; y := lag;
   end if;
```

$\{ \; x \geq y \geq z \; \}$

Soluzioa: Bai, zuzena da. Frogapena:

1. $\{ \; true \; \wedge \; x < y \; \} \equiv \{ \; x < y \; \}$
   `lag := x;`
   $\{ \; lag < y \; \}$ **(AA)**

2. $\{ \; lag < y \; \}$
   `x := y;`
   $\{ \; lag < x \; \}$ **(AA)**

3. $\{ \; lag < x \; \}$
   `y := lag;`
   $\{ \; y < x \; \}$ **(AA)**

4. $(\; y < x \;) \; \rightarrow \; (\; x \geq y \;)$

5. $\{ \; x < y \; \}$
   `lag := x;`
   `x := y;`
   `y := lag;`
   $\{ \; x \geq y \; \}$ 1, 2, 3, 4, **(KPE)** eta **(ODE)**

6. $(\; true \; \wedge \; x \geq y \;) \; \rightarrow \; (\; x \geq y \;)$

7. $(\; true \;) \; \rightarrow \; def(x < y)$

8. $\{ \; true \; \}$
   ```
   if x < y then
      lag := x;
      x := y;
      y := lag;
   end if;
   ```
   $\{ \; x \geq y \; \}$ 5, 6, 7 eta **(BDE)**

9. $\{ \; x \geq y \; \wedge \; y < z \; \}$
   `lag := y;`
   $\{ \; x \geq lag \; \wedge \; lag < z \; \}$ **(AA)**

10. $\{ \; x \geq lag \; \wedge \; lag < z \; \}$
   `y := z;`
   $\{ \; x \geq lag \; \wedge \; lag < y \; \}$ **(AA)**

11. $\{ x \geq lag \wedge lag < y \}$

```
z := lag;
```

$\{ x \geq z \wedge z < y \}$ **(AA)**

12. $(x \geq z \wedge z < y) \rightarrow (x \geq z \wedge y \geq z)$

13. $\{ x \geq z \wedge y < z \}$

```
lag := y;
y := z;
z := lag;
```

$\{ x \geq z \wedge y \geq z \}$ 9, 10, 11, 12, **(KPE)** eta **(ODE)**

14. $(x \geq z \wedge y \geq z) \rightarrow (x \geq z \wedge y \geq z)$

15. $(x \geq y) \rightarrow def(y < z)$

16. $\{ x \geq z \}$

```
if y < z then
   lag := y;
   y := z;
   z := lag;
end if;
```

$\{ x \geq z \wedge y \geq z \}$ 13, 14, 15 eta **(BDE)**

17. $\{ x \geq z \wedge y \geq z \wedge x < y \}$

```
lag := x;
```

$\{ lag \geq z \wedge y \geq z \wedge lag < y \}$ **(AA)**

18. $\{ lag \geq z \wedge y \geq z \wedge lag < y \}$

```
x := y;
```

$\{ lag \geq z \wedge x \geq z \wedge lag < x \}$ **(AA)**

19. $\{ lag \geq z \wedge x \geq z \wedge lag < x \}$

```
y := lag;
```

$\{ y \geq z \wedge x \geq z \wedge y < x \}$ **(AA)**

20. $(y \geq z \wedge x \geq z \wedge y < x) \rightarrow (x \geq y \geq z)$

21. $\{ y \geq z \wedge x \geq z \wedge x < y \}$

```
lag := x;
x := y;
y := lag;
```

$\{ x \geq y \geq z \}$ 17, 18, 19, 20, **(KPE)** eta **(ODE)**

22. $(y \geq z \wedge x \geq z \wedge x \geq y) \rightarrow (x \geq z \wedge y \geq z)$

23. $(x \geq z \wedge y \geq z) \rightarrow def(x < y)$

24. $\{ x \geq z \wedge y \geq z \}$

```
if x < y then
   lag := x;
   x := y;
   y := lag;
end if;
```

$\{ x \geq y \geq z \}$ 21, 22, 23 eta **(BDE)**

25. $\{ \; true \; \}$

```
if x < y then
   lag := x; x := y; y := lag;
end if;
if y < z then
   lag := y; y := z; z := lag;
end if;
if x < y then
   lag := x; x := y; y := lag;
end if;
```

$\{ \; x \geq y \geq z \; \}$ 8, 16, 24 eta **(KPE)**

8. Asmatu inferentzi erregela egokia honako aginduentzat:

8.1.

```
if B1 then I1;
elsif B2 then I2;
   ...
elsif Bn then In;
else In+1;
end if;
```

Soluzioa:

(BDE)

$$\frac{\begin{array}{l} \{\; \phi \wedge B_1 \;\}\; \mathtt{I_1} \;\{\; \psi \;\}, \{\; \phi \wedge \neg B_1 \wedge B_2 \;\}\; \mathtt{I_2} \;\{\; \psi \;\}, \\ \ldots, \{\; \phi \wedge \neg B_1 \wedge \ldots \wedge \neg B_{n-1} \wedge B_n \;\}\; \mathtt{I_n} \;\{\; \psi \;\}, \\ \{\; \phi \wedge \neg B_1 \wedge \ldots \wedge \neg B_{n-1} \wedge \neg B_n \;\}\; \mathtt{I_{n+1}} \;\{\; \psi \;\}, \\ (\; \phi \rightarrow def(B_1)\;), (\;(\; \phi \wedge \neg B_1 \;) \rightarrow def(B_2)\;), \\ \ldots, (\;(\; \phi \wedge \neg B_1 \wedge \ldots \wedge \neg B_{n-1} \;) \rightarrow def(B_n)\;) \end{array}}{\begin{array}{l} \mathtt{if\; B_1\; then\; I_1;} \\ \mathtt{elsif\; B_2\; then\; I_2;} \\ \quad \ldots \\ \mathtt{elsif\; B_n\; then\; I_n;} \\ \mathtt{else\; I_{n+1};} \\ \mathtt{end\; if;} \end{array}}$$

Kasu-hautaketa aginduen aldaera desberdinak dira honako hauek. E datu-mota diskretu bateko espresioa da, eta b_i balioak berekoak dira (bere artean desberdinak).

8.2.

```
case E is
   when b1 => I1;
      ...
   when bn => In;
end case;
```

P_i exekutatzen da E b_i denean. E ebaluatzean $b_1, \ldots, b_n$ ez den balioa lortzen bada, ez da ezer exekutatzen.

Soluzioa:

(BDE)

$$\frac{\begin{array}{l}\{\ \phi\ \wedge\ E=b_1\ \}\ \mathtt{I_1}\ \{\ \psi\ \},\ \{\ \phi\ \wedge\ E\neq b_1\ \wedge\ E=b_2\ \}\ \mathtt{I_2}\ \{\ \psi\ \},\\ \ldots,\ \{\ \phi\ \wedge\ E\neq b_1\ \wedge\ \ldots\ \wedge\ E\neq b_{n-1}\ \wedge\ E=b_n\ \}\ \mathtt{I_n}\ \{\ \psi\ \},\\ (\ \phi\ \wedge\ E\neq b_1\ \wedge\ \ldots\ \wedge\ E\neq b_{n-1}\ \wedge\ E\neq b_n\ )\ \rightarrow\ (\ \psi\ )\\ (\ \phi\ \rightarrow\ def(E=b_1)\ ),\ (\ (\ \phi\ \wedge\ E\neq b_1\ )\ \rightarrow\ def(E=b_2)\ ),\\ \ldots,\ (\ (\ \phi\ \wedge\ E\neq b_1\ \wedge\ \ldots\ \wedge\ E\neq b_{n-1}\ )\ \rightarrow\ def(E=b_n)\ )\end{array}}{\begin{array}{l}\underline{\mathtt{case}}\ \mathtt{E}\ \underline{\mathtt{is}}\\ \quad \underline{\mathtt{when}}\ \mathtt{b_1}\ \mathtt{=>}\ \mathtt{I_1;}\\ \qquad \ldots\\ \quad \underline{\mathtt{when}}\ \mathtt{b_n}\ \mathtt{=>}\ \mathtt{I_n;}\\ \underline{\mathtt{end\ case}};\end{array}}$$

8.3.

```
case E is
   when b1 => I1;
      ...
   when bn => In;
   when others => In+1;
end case;
```

P_i exekutatzen da E b_i denean. E ebaluatzean $b_1, \ldots, b_n$ ez den balioa lortzen bada, P_{n+1} exekutatzen da.

Soluzioa:

(BDE)

$$\frac{\begin{array}{l}\{\ \phi\ \wedge\ E=b_1\ \}\ \mathtt{I_1}\ \{\ \psi\ \},\ \{\ \phi\ \wedge\ E\neq b_1\ \wedge\ E=b_2\ \}\ \mathtt{I_2}\ \{\ \psi\ \},\\ \ldots,\ \{\ \phi\ \wedge\ E\neq b_1\ \wedge\ \ldots\ \wedge\ E\neq b_{n-1}\ \wedge\ E=b_n\ \}\ \mathtt{I_n}\ \{\ \psi\ \},\\ \{\ \phi\ \wedge\ E\neq b_1\ \wedge\ \ldots\ \wedge\ E\neq b_{n-1}\ \wedge\ E\neq b_n\ \}\ \mathtt{I_{n+1}}\ \{\ \psi\ \},\\ (\ \phi\ \rightarrow\ def(E=b_1)\ ),\ (\ (\ \phi\ \wedge\ E\neq b_1\ )\ \rightarrow\ def(E=b_2)\ ),\\ \ldots,\ (\ (\ \phi\ \wedge\ E\neq b_1\ \wedge\ \ldots\ \wedge\ E\neq b_{n-1}\ )\ \rightarrow\ def(E=b_n)\ )\end{array}}{\begin{array}{l}\underline{\mathtt{case}}\ \mathtt{E}\ \underline{\mathtt{is}}\\ \quad \underline{\mathtt{when}}\ \mathtt{b_1}\ \mathtt{=>}\ \mathtt{I_1;}\\ \qquad \ldots\\ \quad \underline{\mathtt{when}}\ \mathtt{b_n}\ \mathtt{=>}\ \mathtt{I_n;}\\ \quad \underline{\mathtt{when\ others}}\ \mathtt{=>}\ \mathtt{I_{n+1};}\\ \underline{\mathtt{end\ case}};\end{array}}$$