Soluciones a los problemas de dualidad

1. Cálculo de modelos duales.

1.1. $\max \ G = y_1 + 7y_2 + 10y_3$

sujeto a

$$\begin{aligned} y_1 + 2y_2 + y_3 &\geq 2 \\ 2y_1 - 2y_2 + 2y_3 &\leq 3 \\ 5y_1 + 4y_2 + y_3 &= -4 \\ y_1, y_3 \geq 0, y_2 &: \text{no rest.} \end{aligned}$$

1.2. $\max \ G = -7y_1 + 12y_2 + 5y_3$

sujeto a

$$\begin{aligned} 4y_1 + 2y_2 + 2y_3 &\leq 1 \\ -y_1 - 4y_2 + 8y_3 &\leq 3 \\ 2y_1 + 4y_3 &\leq 1 \\ y_1 \leq 0, y_2, y_3 &\geq 0 \end{aligned}$$

1.3. $\min \ G = 12y_1 - 8y_2 + 10y_3$

sujeto a

$$\begin{aligned} 2y_1 - y_2 + 3y_3 &\leq 2 \\ y_1 + 5y_2 + 4y_3 &\geq 2 \\ 2y_1 - 2y_2 - 6y_3 &\geq 5 \\ y_1 : \text{no rest.}, y_2 \leq 0, y_3 &\geq 0 \end{aligned}$$

1.4. $\min \ G = -4y_1 + 2y_2 + 6y_3$

sujeto a

$$\begin{aligned} y_1 - y_2 + 4y_3 &\geq 1 \\ y_1 + 6y_2 - y_3 &\geq 1 \\ 2y_1 + 2y_2 + y_3 &= 5 \\ y_1 \geq 0, y_2 \leq 0, y_3 &: \text{no rest.} \end{aligned}$$

1.5. $\max \ G = -6y_1 + 6y_2 + 10y_3$

sujeto a

$$\begin{aligned} 4y_1 + y_2 + 5y_3 &\geq 4 \\ -2y_1 + y_2 + 2y_3 &\geq 1 \\ 3y_1 + y_2 - y_3 &\leq -1 \\ y_1 + y_2 - y_3 &\leq 2 \\ y_1 \leq 0, y_2 : \text{no rest.}, y_3 &\geq 0 \end{aligned}$$

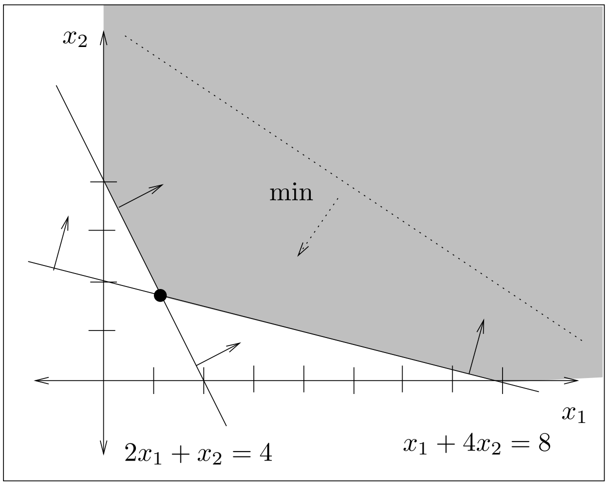
1.6. $\min \ G = 14y_1 - 6y_2 + 10y_3 + 3y_4$

sujeto a

$$\begin{aligned} 2y_1 - y_2 + 4y_3 + y_4 &\geq 1 \\ -4y_1 + 8y_2 + 6y_3 + 9y_4 &\leq 4 \\ y_1, y_3 \geq 0, y_2 \leq 0, y_4 &: \text{no rest.} \end{aligned}$$

2. Soluciones gráficas del modelo lineal y su correspondiente dual.

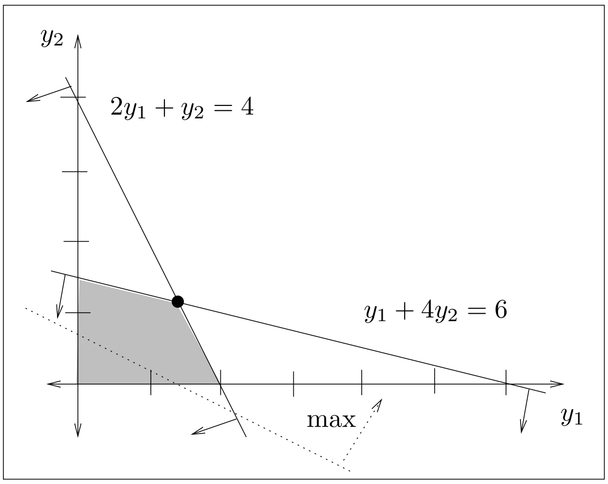
   2.1 Primal. Solucion óptima única: $x_1^* = \frac{8}{7},\ x_2^* = \frac{12}{7},\ z^* = \frac{104}{7}$.



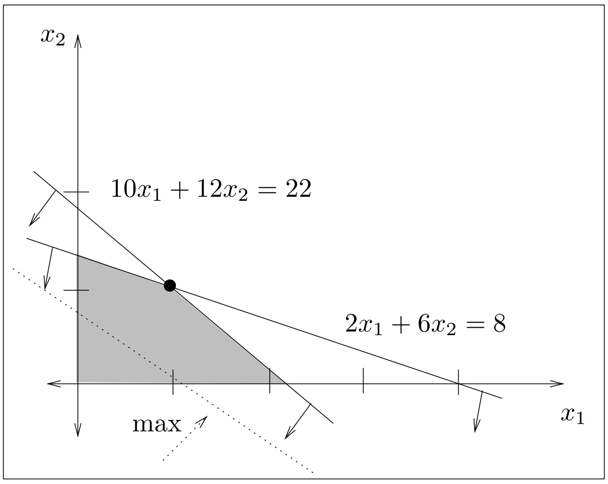
El problema dual.

$$
\begin{aligned}
&\max\ G = 4y_1 + 8y_2 \\
&\text{sujeto a} \\
&\qquad 2y_1 + y_2 \leq 4 \\
&\qquad y_1 + 4y_2 \leq 6 \\
&\qquad y_1, y_2 \geq 0
\end{aligned}
$$

Solución óptima única: $y_1^* = \frac{10}{7},\ y_2^* = \frac{8}{7},\ G^* = \frac{104}{7}$.



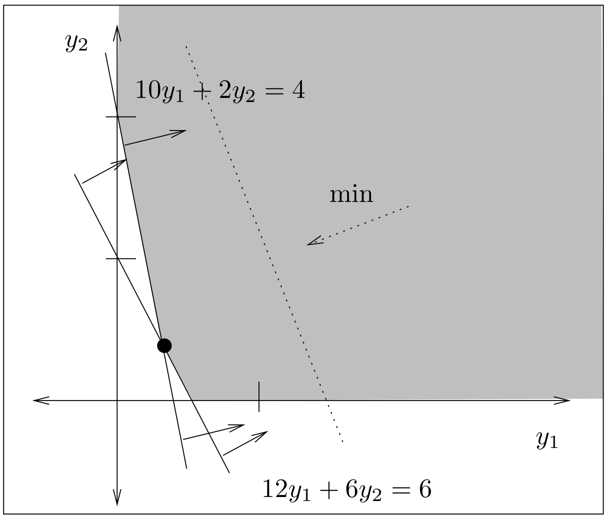
2.2 Primal. Solución óptima única: $x_1^* = 1,\ x_2^* = 1,\ z^* = 10$.



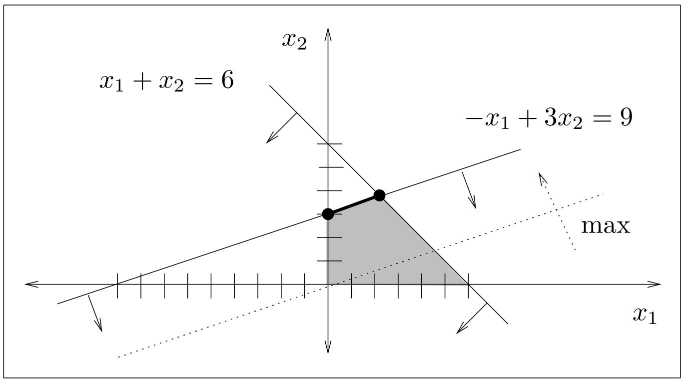
El problema dual.

$$\begin{aligned}
&\min\ G = 22y_1 + 8y_2\\
&\text{sujeto a}\\
&\qquad 10y_1 + 2y_2 \geq 4\\
&\qquad 12y_1 + 6y_2 \geq 6\\
&\qquad\qquad y_1, y_2 \geq 0
\end{aligned}$$

Solución óptima única: $y_1^* = \frac{1}{3},\ y_2^* = \frac{1}{3},\ G^* = 10$.



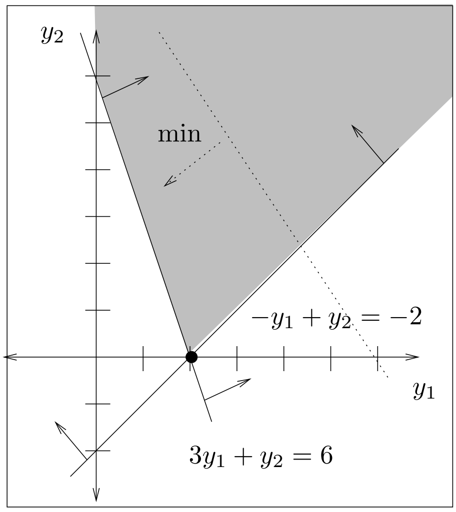
2.3 Primal. Soluciones óptimas múltiples; son soluciones óptimas los puntos $x_1^* = 0,\ x_2^* = 3$ y $x_1^* = \frac{9}{4},\ x_2^* = \frac{15}{4}$ y el segmento que los une. Para todos los puntos $z^* = 18$.



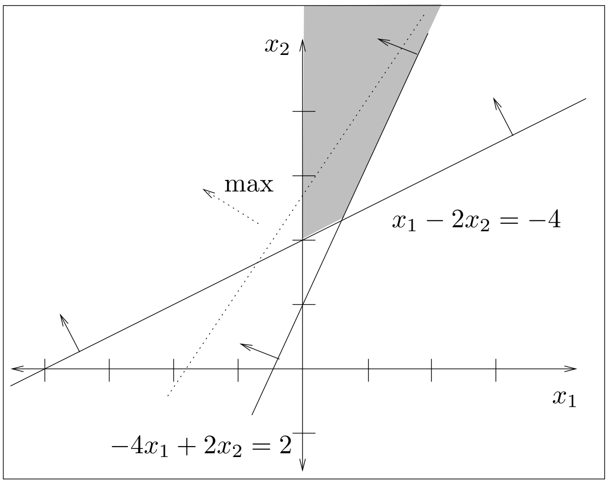
El problema dual.

$$\begin{aligned} &\min\ G = 9y_1 + 6y_2 \\ &\text{sujeto a} \\ &\qquad -y_1 + y_2 \geq -2 \\ &\qquad 3y_1 + y_2 \geq 6 \\ &\qquad y_1, y_2 \geq 0 \end{aligned}$$

Solución óptima única: $y_1^* = 2,\ y_2^* = 0,\ G^* = 18$.



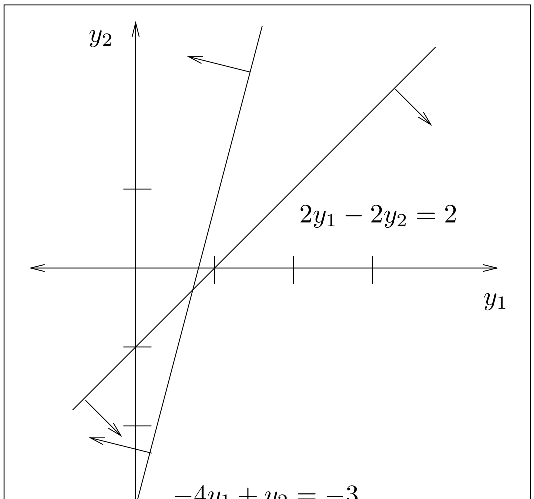
2.4 Primal no acotado.



El problema dual.

$$
\begin{aligned}
&\min \ G = 2y_1 - 4y_2 \\
&\text{sujeto a} \\
&\quad -4y_1 + y_2 \geq -3 \\
&\quad\ \ 2y_1 - 2y_2 \geq 2 \\
&\quad\ \ y_1 \leq 0, y_2 \geq 0
\end{aligned}
$$

El dual es infactible.



3. Solución de modelos utilizando el algoritmo simplex dual.

   3.1 Solución optima única:

$$x_1^* = 4, \quad x_2^* = 0, \quad x_3^* = 0, \quad z^* = -8.$$

   3.2 Solución óptima única:

$$x_1^* = 0, \quad x_2^* = 3, \quad x_3^* = 8, \quad x_4^* = 0, \quad z^* = 27.$$

   3.3 Soluciones óptimas múltiples: $z^* = -30$.

   3.4 Problema infactible.

   3.5 Problema infactible.

   3.6 Solución óptima única:

$$x_1^* = 0, \quad x_2^* = 6, \quad x_3^* = 0, \quad x_4^* = 12, \quad z^* = 84.$$

   3.7 Solución óptima única:

$$x_1^* = 11, \quad x_2^* = 0, \quad x_3^* = 1, \quad x_4^* = 0, \quad z^* = 35.$$

   3.8 Solución no acotada.

   3.9 Solución no acotada.

4.

   4.1 El modelo dual.

$$\begin{aligned}
&\min\ G = 2y_1 + 3y_2 + 3y_3 + 2y_4 \\
&\text{sujeto a} \\
&\qquad y_1 + 2y_2 + 2y_3 + 4y_4 \geq 10 \\
&\qquad 2y_1 + y_2 + 2y_3 + y_4 \geq 6 \\
&\qquad y_1, y_2, y_3, y_4 \geq 0
\end{aligned}$$

   4.2 Solución del modelo dual utilizando el algoritmo simplex dual.

$$y_1^* = 2, \quad y_2^* = 0, \quad y_3^* = 0, \quad y_4^* = 2, \quad G^* = 8.$$

   4.3 Solución óptima del primal obtenida de la tabla óptima del dual.

$$x_1^* = \frac{2}{7}, \quad x_2^* = \frac{6}{7}, \quad z^* = 8.$$

5.

   5.1 El modelo dual.

$$\begin{aligned}
&\max\ G = 20y_1 + 16y_2 + 18y_3 + 21y_4 \\
&\text{sujeto a} \\
&\qquad 4y_1 + 6y_2 + 4y_3 + 4y_4 \leq 30 \\
&\qquad 2y_1 + 4y_2 + 2y_3 + 4y_4 \leq 28 \\
&\qquad y_1, y_2, y_3, y_4 \geq 0
\end{aligned}$$

5.2 Solución del modelo dual utilizando el algoritmo simplex primal.

$$y_1^* = 1, \;\; y_2^* = 0, \;\; y_3^* = 0, \;\; y_4^* = \frac{13}{2}, \;\; G^* = \frac{313}{2}.$$

5.3 Solución óptima del primal obtenida de la tabla óptima del dual.

$$x_1^* = \frac{19}{4}, \quad x_2^* = \frac{1}{2}, \quad z^* = \frac{313}{2}.$$

6.

6.1 (a) Solución óptima del modelo.

$$x_1^* = \frac{7}{2}, \quad x_2^* = \frac{3}{2}, \quad x_3^* = 0, \quad z^* = \frac{57}{2}.$$

(b) El modelo dual

$$\begin{aligned} &\min \; G = 90y_1 + 60y_2 \\ &\text{sujeto a} \\ &\qquad 15y_1 + 15y_2 \geq 6 \\ &\qquad 25y_1 + 5y_2 \geq 5 \\ &\qquad 30y_1 + 15y_2 \geq 4 \\ &\qquad y_1, y_2 \geq 0 \end{aligned}$$

Solución óptima del dual $y_1^* = \frac{3}{20}, \;\; y_2^* = \frac{1}{4}, \;\; G^* = \frac{57}{2}.$

(c) Precios sombra.

$$\text{Recurso } b_1 \rightarrow \overset{\wedge}{\mathbf{b}} = \begin{pmatrix} 91 \\ 60 \end{pmatrix}, \quad \mathbf{x}_B = \mathbf{B}^{-1} \overset{\wedge}{\mathbf{b}} \geq \mathbf{0}.$$

$y_1^* = \frac{3}{20}$ es el precio sombra del recurso b_1.
$\hat{z} = z + y_1^* = \frac{57}{2} + \frac{3}{20}.$

$$\text{Recurso } b_2 \rightarrow \overset{\wedge}{\mathbf{b}} = \begin{pmatrix} 90 \\ 61 \end{pmatrix}, \quad \mathbf{x}_B = \mathbf{B}^{-1} \overset{\wedge}{\mathbf{b}} \geq \mathbf{0}.$$

$y_2^* = \frac{1}{4}$ es el precio sombra del recurso b_2.
$\hat{z} = z + y_2^* = \frac{57}{2} + \frac{1}{4}.$

6.2 (a) Solución óptima del modelo.

$$x_1^* = 12, \quad x_2^* = 0, \quad x_3^* = 0 \quad z^* = 24.$$

(b) El modelo dual.

$$\begin{aligned} \min\ G = 12y_1 + 8y_2 \\ \text{sujeto a} \\ y_1 + 4y_2 \geq 2 \\ 2y_1 + 2y_2 \geq 1 \\ 4y_1 \geq -1 \\ y_1 \geq 0, y_2 \leq 0 \end{aligned}$$

Solución óptima del dual. $y_1^* = 2, \quad y_2^* = 0, \quad G^* = 24.$

(c) Precios sombra.

$$\text{Recurso } b_1 \rightarrow \hat{\mathbf{b}} = \begin{pmatrix} 13 \\ 8 \end{pmatrix}, \quad \mathbf{x}_B = \mathbf{B}^{-1}\,\hat{\mathbf{b}} \geq \mathbf{0}.$$

$y_1^* = 2$ es el precio sombra del recurso b_1.
$\hat{z} = z + y_1^* = 24 + 2.$

$$\text{Recurso } b_2 \rightarrow \hat{\mathbf{b}} = \begin{pmatrix} 12 \\ 9 \end{pmatrix}, \quad \mathbf{x}_B = \mathbf{B}^{-1}\,\hat{\mathbf{b}} \geq \mathbf{0}.$$

$y_2^* = 0$ es el precio sombra del recurso b_2.
$\hat{z} = z + y_2^* = 24 - 0.$