The transportation problem and the assignment problem. Solutions

1. The matrix format of the transportation problem where the objective is to minimize the company's transportation cost:

	C_1	C_2	C_3	Supply
P_1	100	100	50	15
P_2	650	110	100	15
P_3	60	65	75	15
P_4	150	90	70	15
Demand	30	16	14	

2. The matrix format of the transportation problem where the objective is to maximize:

	S_1	S_2	S_3	Supply
P_1	26	13	22	100
P_2	30	21	27	85
P_3	34	22	30	140
P_4	25	18	24	125
Demand	125	150	175	

3. The matrix format of the transportation problem where the objective is to maximize the total benefit:

	C_1	C_2	C_3	C_4	Supply
P_1	60	40	45	55	130
P_2	70	55	65	60	200
P_3	80	60	55	75	170
P_4	$-M$	$-M$	0	0	50
Demand	150	175	175	50	

4. The matrix format of the transportation problem where the objective is to minimize:

	1st week	2nd week	3rd week	Supply
Stored in the warehouse	0	15	30	2
1st week, regular-time shift	20	35	50	5
1st week, extended shift	30	45	60	5
2nd week, regular-time shift	M	30	45	4
2nd week, extended shift	M	40	55	5
3rd week, regular-time shift	M	M	45	2
3rd week, extended shift	M	M	55	5
Demand	8	8	8	

5. The initial basic feasible solutions.

   5.1 Applying the northwest corner method:
   $x_{11} = 300$, $x_{12} = 100$, $x_{22} = 200$, $x_{32} = 40$, $x_{33} = 400$, $x_{34} = 180$, $z = 14280$.
   Applying Vogel's approximation method:
   $x_{14} = 400$, $x_{21} = 200$, $x_{31} = 100$, $x_{32} = 340$, $x_{33} = 140$, $x_{34} = 40$, $z = 11420$.

   5.2 Applying the northwest corner method, the solution is degenerate:
   $x_{11} = 10$, $x_{12} = 10$, $x_{13} = 10$, $x_{23} = 10$, $x_{24} = 20$, $x_{35} = 30$, $z = 3680$.
   Applying Vogel's approximation method:
   $x_{14} = 10$, $x_{15} = 20$, $x_{21} = 10$, $x_{22} = 10$, $x_{25} = 10$, $x_{33} = 20$, $x_{34} = 10$, $z = 2600$.

   5.3 Applying the northwest corner method:
   $x_{11} = 80$, $x_{21} = 20$, $x_{22} = 80$, $x_{32} = 20$, $x_{33} = 50$, $x_{34} = 5$, $x_{44} = 45$, $x_{45} = 75$, $x_{55} = 25$, $x_{56} = 35$, $x_{66} = 65$, $z = 10585$.
   Applying Vogel's approximation method, the solution is degenerate:
   $x_{13} = 50$, $x_{16} = 30$, $x_{21} = 100$, $x_{35} = 75$, $x_{42} = 40$, $x_{45} = 10$, $x_{46} = 70$, $x_{52} = 60$, $x_{64} = 50$, $x_{65} = 15$, $z = 7300$.

6. The optimal solutions and the minimum transportation costs:

   6.1 $x^*_{11} = 32$, $x^*_{21} = 20$, $x^*_{22} = 3$, $x^*_{32} = 30$, $x^*_{41} = 18$, $x^*_{43} = 22$, $x^*_{44} = 7$, $z^* = 1931$.

   6.2 $x^*_{11} = 20$, $x^*_{14} = 10$, $x^*_{23} = 10$, $x^*_{24} = 2$, $x^*_{32} = 4$, $x^*_{34} = 1$, $x^*_{44} = 10$, $z^* = 782$.

   6.3 Multiple optimal solutions, $z^* = 174$.
   $x^*_{11} = 3$, $x^*_{13} = 7$, $x^*_{24} = 3$, $x^*_{25} = 9$, $x^*_{31} = 1$, $x^*_{32} = 5$, $x^*_{34} = 6$.
   $x^*_{12} = 3$, $x^*_{13} = 7$, $x^*_{24} = 3$, $x^*_{25} = 9$, $x^*_{31} = 4$, $x^*_{32} = 2$, $x^*_{34} = 6$.
   $x^*_{11} = 4$, $x^*_{13} = 6$, $x^*_{23} = 1$, $x^*_{24} = 2$, $x^*_{25} = 9$, $x^*_{32} = 5$, $x^*_{34} = 7$.

6.4 Multiple optimal solutions, $z^* = 199$.

$x^*_{15} = 4$, $x^*_{23} = 6$, $x^*_{31} = 3$, $x^*_{32} = 4$, $x^*_{33} = 1$, $x^*_{34} = 1$.

$x^*_{13} = 1$, $x^*_{15} = 3$, $x^*_{23} = 6$, $x^*_{31} = 3$, $x^*_{32} = 4$, $x^*_{34} = 2$.

6.5 The optimal solution is degenerate, $z^* = 3784$.

$x^*_{14} = 42$, $x^*_{22} = 40$, $x^*_{31} = 18$, $x^*_{32} = 10$, $x^*_{33} = 8$, $x^*_{35} = 12$, $x^*_{44} = 10$.

6.6 The optimal solution is degenerate, $z^* = 363$.

$x^*_{12} = 10$, $x^*_{13} = 5$, $x^*_{22} = 5$, $x^*_{31} = 9$, $x^*_{34} = 3$, $x^*_{44} = 2$, $x^*_{51} = 6$.

7. The optimal assignment: $A_1 \to D$, $A_2 \to C$, $A_3 \to A$, $A_4 \to B$. Applicant A_5 remains unemployed. $z^* = 25$.

8. Multiple optimal assignments, $z^* = 30$.

   $T_1 \to P_2$, $T_2 \to P_4$, $T_3 \to P_1$, $T_4 \to P_3$. The production plant P_5 does not receive any truck.

   $T_1 \to P_2$, $T_2 \to P_4$, $T_3 \to P_3$, $T_4 \to P_5$. The production plant P_1 does not receive any truck.

9. The optimal assignments and z^*:

   9.1 $O_1 \to D_4$, $O_2 \to D_2$, $O_3 \to D_1$, $O_4 \to D_3$, $z^* = 27$.

   9.2 $O_1 \to D_2$, $O_2 \to D_4$, $O_3 \to D_5$, $O_4 \to D_1$, $O_5 \to D_3$, $z^* = 29$.

   9.3 $O_1 \to D_1$, $O_2 \to D_2$, $O_3 \to D_5$, $O_4 \to D_4$, $O_5 \to D_3$, $z^* = 55$.