Linear Modeling and Graphical Solution. Solutions

Linear Modeling

1. The jam production problem.

   Decision variables:

   x_i= number of kg of type i one flavour jam to be produced, $i = 1$ (apple), 2 (plum), 3 (peach).

   y_i= number of kg of type i two flavours jam to be produced, $i = 1$ (apple+plum), 2 (apple+peach).

   Linear model:

$$
\begin{aligned}
\max\ z = (2-0.4)x_1 + (2-0.6)x_2 + (2-0.8)x_3 + \\
+(2.5-0.4)\frac{1}{2}(y_1+y_2) + (2.5-0.6)\frac{1}{2}y_1 + (2.5-0.8)\frac{1}{2}y_2 \\
\text{subject to} \\
x_1 \geq 175 \\
x_2 \geq 160 \\
x_3 \geq 150 \\
x_1 + \frac{1}{2}y_1 + \frac{1}{2}y_2 \leq 1000 \\
x_2 + \frac{1}{2}y_1 \leq 600 \\
x_3 + \frac{1}{2}y_2 \leq 800 \\
x_1, x_2, x_3, y_1, y_2 \geq 0
\end{aligned}
$$

2. The fruit salad production problem.

   Decision variables:

   x_i= Quantity of fruit i in one kg of normal fruit salad, $i = 1$ (cherry), 2 (watermelon), 3 (mango), 4 (orange), 5 (melon), 6 (banana).

   y_i= Quantity of fruit i in one kg of low calorie fruit salad, $i = 1$ (cherry), 2 (watermelon), 3 (mango), 4 (orange), 5 (melon), 6 (banana).

   Linear model:

$$\begin{aligned}
\min\ z = 7(x_1+y_1) + 0.9(x_2+y_2) + 4(x_3+y_3) + \\
+1.6(x_4+y_4) + 1.4(x_5+y_5) + 1.5(x_6+y_6)
\end{aligned}$$

subject to

$$\begin{aligned}
250x_1 + 100x_2 + 150x_3 + 400x_4 + 140x_5 + 90x_6 \geq 150 \\
200x_1 + 90x_2 + 220x_3 + 200x_4 + 160x_5 + 280x_6 \geq 200 \\
120x_1 + 60x_2 + 50x_3 + 550x_4 + 300x_5 + 100x_6 \geq 200 \\
700y_1 + 300y_2 + 580y_3 + 490y_4 + 300y_5 + 900y_6 \leq 400 \\
200y_1 + 90y_2 + 220y_3 + 200y_4 + 160y_5 + 280y_6 \geq 100 \\
120y_1 + 60y_2 + 50y_3 + 550y_4 + 300y_5 + 100y_6 \geq 250 \\
x_1 + x_2 \geq 0.1(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) \\
y_1 + y_2 \geq 0.1(y_1 + y_2 + y_3 + y_4 + y_5 + y_6) \\
x_3 + x_4 \geq 0.3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) \\
y_3 + y_4 \geq 0.3(y_1 + y_2 + y_3 + y_4 + y_5 + y_6) \\
x_5 + x_6 \geq 0.2(x_1 + x_2 + x_3 + x_4 + x_5 + x_6) \\
y_5 + y_6 \geq 0.2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6) \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1 \\
y_1 + y_2 + y_3 + y_4 + y_5 + y_6 = 1 \\
x_i, y_i \geq 0, \quad i = 1,2,3,4,5,6
\end{aligned}$$

3. The problem of the summer camps.

Decision variables:

x_{ij}= number of girls whose mother tongue is i will go to the summer camp S_j, $i = b$ (Basque), s (Spanish), $j = 1, 2$.

y_{ij}= number of boys whose mother tongue is i will go to the summer camp S_j, $i = b$ (Basque), s (Spanish), $j = 1, 2$.

Linear model:

$$
\begin{aligned}
\min\ z = 8x_{b1} + 8y_{b1} + 8x_{s1} + 8y_{s1} + 26x_{b2} + 26y_{b2} + 26x_{s2} + 26y_{s2} \\
\text{subject to} \qquad\qquad\qquad\qquad \\
x_{b1} + x_{b2} = 650 \\
y_{b1} + y_{b2} = 600 \\
x_{s1} + x_{s2} = 475 \\
y_{s1} + y_{s2} = 475 \\
x_{b1} + y_{b1} + x_{s1} + y_{s1} <= 800 \\
x_{b1} + y_{b1} \geq 0.5(x_{b1} + y_{b1} + x_{s1} + y_{s1}) \\
x_{b2} + y_{b2} \geq 0.5(x_{b2} + y_{b2} + x_{s2} + y_{s2}) \\
x_{b1} + x_{s1} \geq 0.5(x_{b1} + y_{b1} + x_{s1} + y_{s1}) \\
x_{b2} + x_{s2} \geq 0.5(x_{b2} + y_{b2} + x_{s2} + y_{s2}) \\
x_{b1}, x_{b2}, x_{s1}, x_{s2} \geq 0 \quad \text{and integer} \\
y_{b1}, y_{b2}, y_{s1}, y_{s2} \geq 0 \quad \text{and integer}
\end{aligned}
$$

4. The problem about the textile enterprise.

Decision variables:

x_{ij}= number of suits of type j fabric produced in month i, $i=1,\ldots,6$, $j=1$ (new), 2 (recycled) .

y_{ij}= number of suits of type j fabric stored in month i, $i=1,\ldots,5$, $j=1$ (new), 2 (recycled) .

Linear model:

$$\begin{aligned}
\min\ z = {} & 70(x_{11}+x_{21}+x_{31}+x_{41}+x_{51}+x_{61})+60(x_{12}+x_{22}+x_{32}+x_{42}+\\
& +x_{52}+x_{62})+y_{11}+y_{21}+y_{31}+y_{41}+y_{51}+y_{12}+y_{22}+y_{32}+y_{42}+y_{52}
\end{aligned}$$

subject to

$$\begin{aligned}
x_{11} &= 100+y_{11}, & x_{12} &= 100+y_{12}\\
x_{21}+y_{11} &= 300+y_{21}, & x_{22}+y_{12} &= 150+y_{22}\\
x_{31}+y_{21} &= 500+y_{31}, & x_{32}+y_{22} &= 300+y_{32}\\
x_{41}+y_{31} &= 600+y_{41}, & x_{42}+y_{32} &= 200+y_{42}\\
x_{51}+y_{41} &= 200+y_{51}, & x_{52}+y_{42} &= 100+y_{52}\\
x_{61}+y_{51} &= 450, & x_{62}+y_{52} &= 300\\
x_{11} &\le 400, & x_{12} &\le 200\\
x_{21} &\le 400, & x_{22} &\le 200\\
x_{31} &\le 400, & x_{32} &\le 200\\
x_{41} &\le 400, & x_{42} &\le 200\\
x_{51} &\le 400, & x_{52} &\le 200\\
x_{61} &\le 400, & x_{62} &\le 200
\end{aligned}$$

$$x_{ij} \ge 0 \text{ and integer },\ i=1,\ldots,6,\ j=1,2$$
$$y_{ij} \ge 0 \text{ and integer },\ i=1,\ldots,5,\ j=1,2$$

5. The inter-school competition problem.

Decision variables:

$$x_i = \begin{cases} 1 & \text{if child } i \text{ is selected to form the group,} \quad i = 1, \ldots, 20. \\ 0 & \text{otherwise.} \end{cases}$$

Linear model:

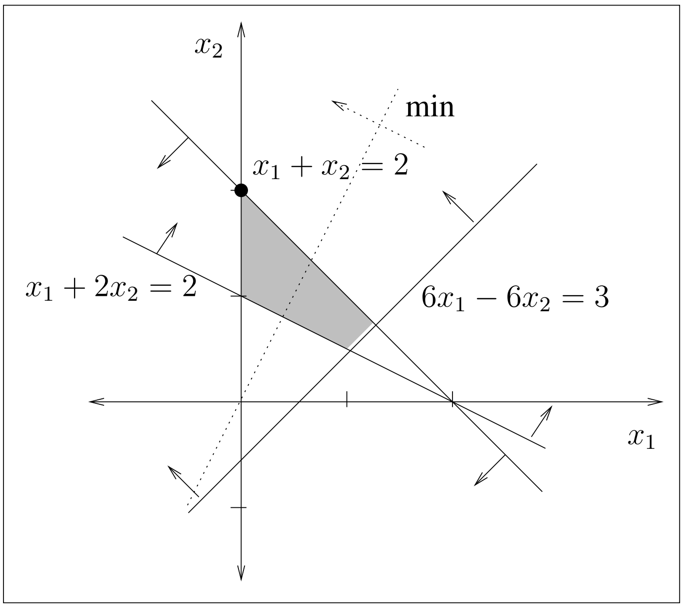
$$\begin{aligned} \min\ z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + \\ +x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} \end{aligned}$$

subject to

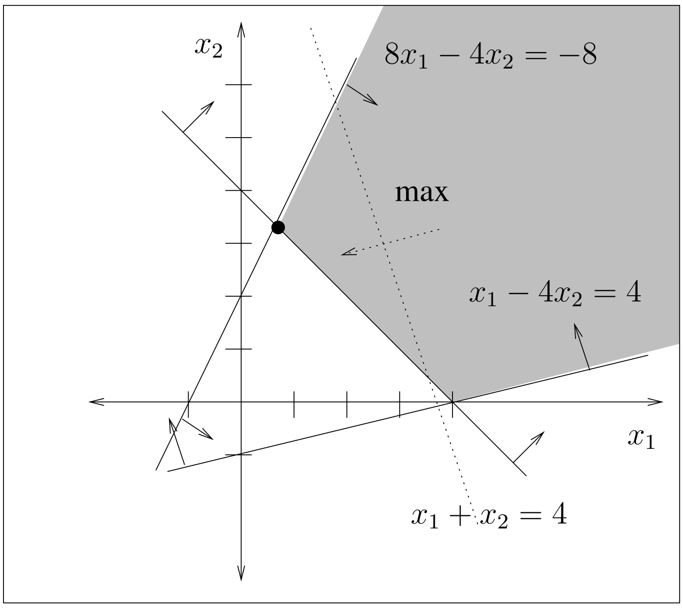
$$\begin{aligned} x_2 + x_6 + x_7 + x_{12} + x_{19} &\geq 1 \\ x_5 + x_{13} + x_{14} &\geq 1 \\ x_9 + x_{15} + x_{18} &\geq 1 \\ x_1 + x_4 + x_9 + x_{10} + x_{20} &\geq 1 \\ x_{13} + x_{17} &\geq 1 \\ x_3 + x_8 + x_{11} + x_{12} + x_{16} &\geq 1 \\ x_1 + x_2 + \cdots + x_{20} &\geq 3 \\ x_i = 0, 1, \quad i = 1, \ldots, &20 \end{aligned}$$

Graphical Solution

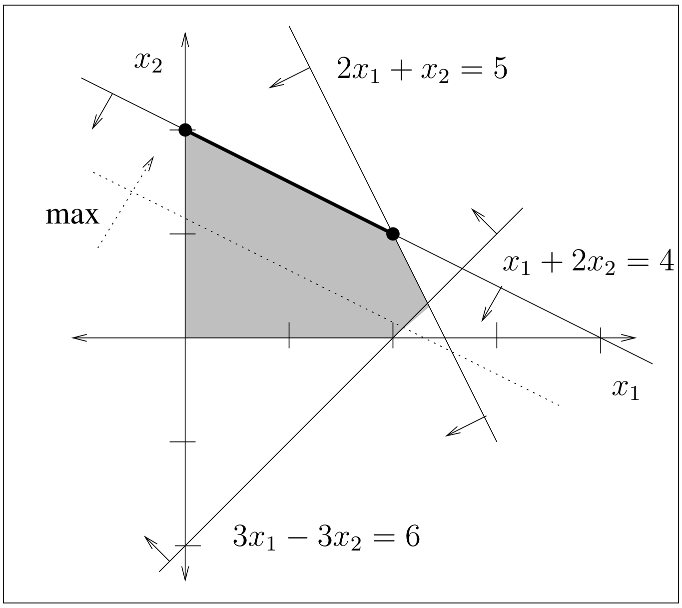
1. There is a unique optimal solution: $x_1^* = 0$, $x_2^* = 2$, $z^* = -2$.



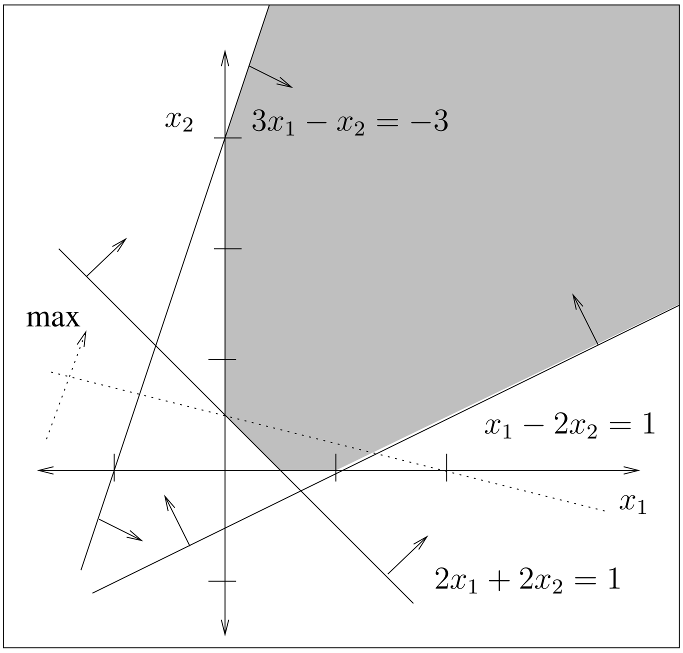
2. There is a unique optimal solution: $x_1^* = \frac{2}{3}$, $x_2^* = \frac{10}{3}$, $z^* = -\frac{32}{3}$.



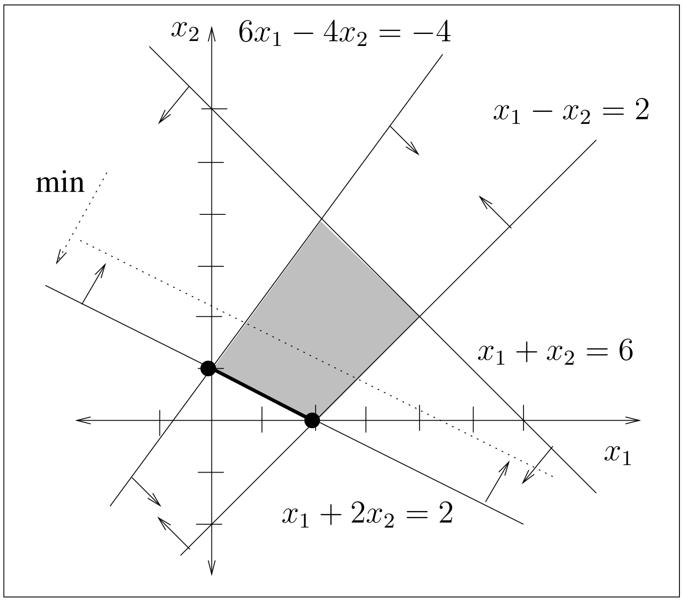
3. There are multiple optimal solutions: the extreme point $x_1^* = 0$, $x_2^* = 2$, the extreme point $x_1^* = 2$, $x_2^* = 1$, and the infinite points lying on the segment line between the two extreme points are all of them optimal solutions to the given linear model. $z^* = 8$.



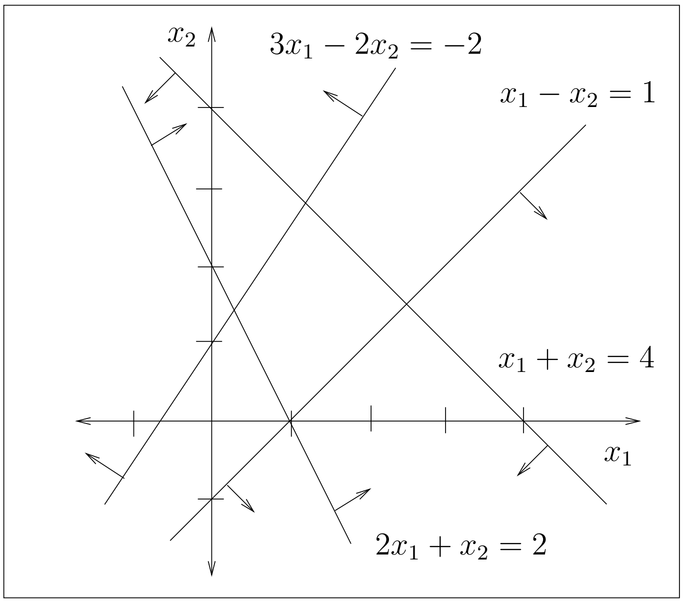
4. Unbounded solution.



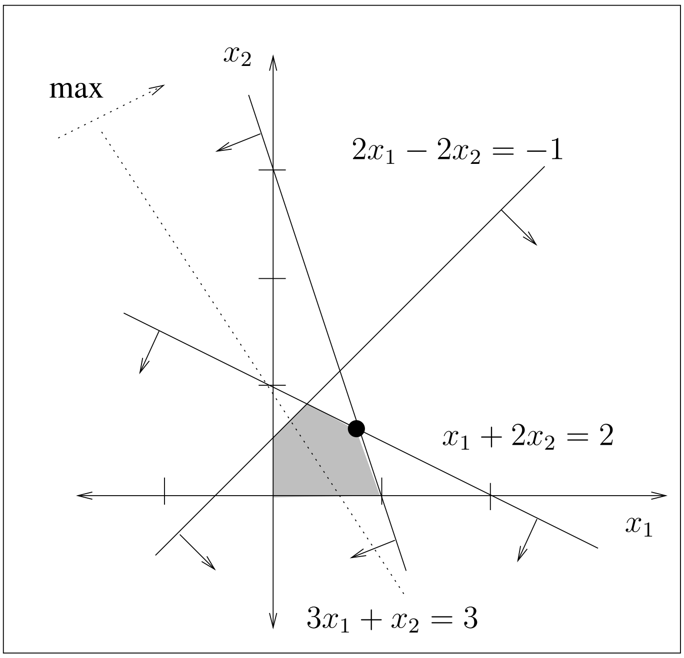
5. There are multiple optimal solutions: the extreme point $x_1^* = 0$, $x_2^* = 1$, the extreme point $x_1^* = 2$, $x_2^* = 0$, and the infinite points lying on the segment line between the two extreme points are all of them optimal solutions to the given linear model. $z^* = 2$.



6. Infeasible problem.



7. There is a unique optimal solution: $x_1^* = \frac{4}{5}$, $x_2^* = \frac{3}{5}$. $z^* = \frac{36}{5}$.



8. Unbounded solution.