Linear programming using Lindo

The aim of this laboratory session is to study in depth the entering vector rule and the leaving vector rule used in the simplex algorithm, using the software tool called Lindo. Lindo gives you the possibility to select the pivot by yourself, if you want to.

The software tool Lindo is widely used to solve optimization models. You can download a trial version free of charge from: http://www.lindo.com, go to "downloads", and there to "Download Classic LINDO". It is available for windows operating system.

1. Analyze the options that Lindo offers you to solve linear models.

2. The simplex algorithm is used when the initial tableau is primal feasible. When the reduced cost coefficients are negative, $z_j - c_j < 0$, the solution may be improved. The entering vector rule and the leaving vector rule are used to change the basis. The process goes on until the optimal solution is found.

   In this exercise you are asked not to follow one of the rules mentioned. You have to show that you understand the consequences that such decision has. Consider the linear model that follows:

$$\begin{aligned} \max\ z = 4x_1 - 3x_2 + x_3 \\ \text{subject to} \qquad\qquad \\ -x_1 + 2x_2 + x_3 \leq 30 \\ 2x_1 + x_2 + x_3 \leq 12 \\ 2x_1 + 3x_2 + x_3 \leq 18 \\ x_1, x_2, x_3 \geq 0 \end{aligned}$$

   2.1 Use lindo to compute the initial tableau. Is it primal feasible? Analyze the tableau and extract the initial basic solution and the objective value.

   2.2 Starting at the initial tableau computed in 2.1, select the entering vector and the leaving vector (go to the "Solve" menu and select the "Pivot" option) according to the following criteria:

   (a) The entering vector $\mathbf{a}_k$ is selected so that $z_k - c_k > 0$, and the leaving vector $\mathbf{a}_r$ is selected so that:

$$\frac{x_{Br}}{y_{rk}} = \min \left\{ \frac{x_{Bi}}{y_{ik}} \ / \ y_{ik} > 0 \right\}$$

   Compute the new tableau and analyze the effect of the wrong substitution performed in the basis.

   (b) The entering vector $\mathbf{a}_k$ is selected so that $z_k - c_k < 0$ but is not the one with the minimum negative reduced cost coefficient. The leaving vector $\mathbf{a}_r$ is selected so that:

$$\frac{x_{Br}}{y_{rk}} = \min \left\{ \frac{x_{Bi}}{y_{ik}} \ / \ y_{ik} > 0 \right\}$$

   Compute the new tableau and analyze the effect of the wrong substitution performed in the basis.

(c) The entering vector $\mathbf{a}_k$ is selected so that:

$$z_k - c_k \ = \ \min \ \{ \ z_j - c_j \ / \ z_j - c_j < 0 \ \}$$

and the leaving vector $\mathbf{a}_r$ is selected so that $y_{rk} < 0$ holds.
Compute the new tableau and analyze the effect of the wrong substitution performed in the basis.

(d) The entering vector $\mathbf{a}_k$ is selected so that:

$$z_k - c_k \ = \ \min \ \{ \ z_j - c_j \ / \ z_j - c_j < 0 \ \}$$

and the leaving vector $\mathbf{a}_r$ is selected so that $y_{rk} > 0$ and the following holds:

$$\frac{x_{Br}}{y_{rk}} \neq \min \left\{ \frac{x_{Bi}}{y_{ik}} \ / \ y_{ik} > 0 \ \right\}$$

Compute the new tableau and analyze the effect of the wrong substitution performed in the basis.

3. Solve the linear model given previously. You can compute the optimal solution directly (go to the "Solve" menu and select the "Solve" option). Lindo will select the entering vector and the leaving vector at each iteration of the algorithm.