The transportation problem and the assignment problem. Exercises

1. A company manufactures a type of product in four different production plants: P_1, P_2, P_3 and P_4. Each of these production plants can produce up to 15 tons per month. The company supplies 30, 16 and 14 tons a month to three customers C_1, C_2 and C_3, respectively.

   The distances measured in km from each production plant to each customer are displayed below:

	C_1	C_2	C_3
P_1	100	100	50
P_2	650	110	100
P_3	60	65	75
P_4	150	90	70

   The cost of transporting each ton of product is of 0.5 euros per km. Formulate the matrix format of the transportation problem so as to minimize the company's transportation cost.

2. A company manufactures a type of product in four different production plants: P_1, P_2, P_3 and P_4. The unit production cost and the production capacity differ from one production plant to another as shown in the table:

Production plant	Production cost	Production capacity
P_1	15	100
P_2	9	85
P_3	7	140
P_4	13	125

   The units produced are sent to three shops S_1, S_2 and S_3. Each shop has a different demand and sells the product unit at a different price as shown in the following table:

Shops	Price	Demand
S_1	45	125
S_2	33	150
S_3	40	175

   The product unit transportation costs are shown below:

	S_1	S_2	S_3
P_1	4	5	3
P_2	6	3	4
P_3	4	4	3
P_4	7	2	3

   Formulate the matrix format of the transportation problem considering that the objective is to maximize.

3. An enterprise manufactures a product in three production plants, P_1, P_2 and P_3, with a production capacity of 130, 200 and 170 units of product, respectively. The demand of four customers has to be satisfied as follows: customer C_1 demands 150 product units, customer C_2 demands 175, and customer C_3 demands at least 125. Both customers C_3 and C_4 are prepared to buy any spare product units, and they both want to buy as many units of product as possible.

   The benefit obtained from the sale of units of product to the customers is the following:

	C_1	C_2	C_3	C_4
P_1	60	40	45	55
P_2	70	55	65	60
P_3	80	60	55	75

   Formulate the matrix format of the transportation problem so as to maximize the total benefit.

4. A production plant aims to schedule production for the next three weeks. Employees work both on regular-time shifts and on extended shifts. 8 machines must be sold every week. The following table shows the production capacity for the following three weeks, both working on regular-time shifts and on extended shifts, and the cost of each working hour.

Week	Production capacity (regular-time shifts)	Production capacity (extended shifts)	Working hour cost (euro)
1	5	5	20
2	4	5	30
3	2	5	45

   The production on extended shifts is more expensive than on regular-time shifts; it costs 10 additional euros. The machines produced and not sold in a week are stored at a holding cost of 15 euros per machine for each extra week that it is stored. There are 2 machines in the warehouse at present, which will be used to satisfy the demand of the next weeks. By the end of the third week, and once the demands have been satisfied, there should not be any machines left in the warehouse.

   The aim is to satisfy the machine demands at a minimum cost. Formulate the matrix format of this transportation problem.

5. Given the following transportation costs tableaux, apply the northwest corner method and Vogel's approximation method to find an initial basic feasible solution.

   5.1

	D_1	D_2	D_3	D_4	Supply
O_1	9	11	11	8	400
O_2	7	12	14	10	200
O_3	11	10	12	16	620
Demand	300	340	400	440	

5.2

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	80	40	60	30	25	30
O_2	50	20	40	35	28	30
O_3	65	50	30	22	26	30
Demand	10	10	20	20	30	

5.3

	D_1	D_2	D_3	D_4	D_5	D_6	Supply
O_1	30	28	12	15	20	10	80
O_2	10	15	12	20	25	10	100
O_3	8	10	6	8	8	10	75
O_4	20	22	24	20	25	21	120
O_5	25	20	30	35	32	28	60
O_6	27	30	25	14	20	26	65
Demand	100	100	50	50	100	100	

6. Consider the following transportation problems in matrix format. Find an initial basic feasible solution using Vogel's approximation method, and apply the transportation algorithm to compute the optimal solution.

6.1

	D_1	D_2	D_3	D_4	Supply
O_1	20	19	10	15	32
O_2	17	15	6	10	23
O_3	18	14	2	6	30
O_4	21	23	3	6	47
Demand	70	33	22	7	

6.2

	D_1	D_2	D_3	D_4	Supply
O_1	15	23	20	25	30
O_2	14	17	11	17	12
O_3	14	7	6	10	5
O_4	8	9	10	5	10
Demand	20	4	10	31	

6.3

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	5	2	3	8	10	10
O_2	7	5	4	5	8	12
O_3	6	3	7	5	9	12
Demand	4	5	7	9	9	

6.4

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	15	14	9	16	11	4
O_2	10	15	8	14	11	6
O_3	13	10	13	15	–	9
Demand	3	4	7	4	6	

6.5

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	32	30	27	26	25	42
O_2	28	25	22	22	19	40
O_3	35	36	29	38	25	48
O_4	20	22	15	17	16	10
Demand	18	50	8	52	12	

6.6

	D_1	D_2	D_3	D_4	Supply
O_1	20	10	5	15	20
O_2	12	8	10	9	5
O_3	11	15	8	9	12
O_4	15	7	15	6	2
O_5	10	20	15	10	6
Demand	15	15	5	5	

7. An enterprise offers 4 new jobs: A, B, C and D. The personnel department has prepared a test and 5 people have applied for it: A_1, A_2, A_3, A_4 and A_5. The aim of the test is to measure to what extent each of the applicants is suitable to perform each of the jobs. The values shown in the table are the number of errors made in the test.

	A	B	C	D
A_1	16	4	17	3
A_2	13	14	8	11
A_3	2	19	–	9
A_4	21	12	13	16
A_5	22	16	25	12

Applicant A_3 is not able to perform job C, and therefore, the assignment must be forbidden. The optimal assignment among applicants and jobs is the one that minimizes the number of errors made in the test.

Use the Hungarian algorithm to find the optimal assignment. According to the optimal assignment found, which of the applicants will remain unemployed?

8. A transportation company has 4 trucks in four different cities: T_1, T_2, T_3 and T_4. Five production plants placed in five cities are demanding a truck: P_1, P_2, P_3, P_4 and P_5. The trucks must be sent to the production plants, so that their production can be delivered. The distances from the cities where the trucks are to the production plants are shown in the table:

	P_1	P_2	P_3	P_4	P_5
T_1	13	1	8	7	10
T_2	12	6	4	4	7
T_3	18	10	14	21	20
T_4	14	13	7	12	11

Use the Hungarian algorithm to find the optimal assignment among trucks and production plants, so that the total distance is minimized. How many different optimal assignments are there? According to each of the optimal assignments, which is the production plant that does not receive any truck?

9. Consider the following assignment costs tableaux. Apply the Hungarian algorithm to find the optimal assignments among the origin points and the destination points.

9.1

	D_1	D_2	D_3	D_4
O_1	10	6	11	10
O_2	18	10	10	16
O_3	2	9	11	4
O_4	11	15	5	15

9.2

	D_1	D_2	D_3	D_4	D_5
O_1	11	4	11	12	17
O_2	7	3	12	5	14
O_3	3	1	9	3	10
O_4	6	9	14	12	15
O_5	13	9	4	13	7

9.3

	D_1	D_2	D_3	D_4	D_5
O_1	23	17	2	27	9
O_2	29	8	3	25	19
O_3	24	34	22	38	5
O_4	13	11	32	15	30
O_5	36	26	4	39	37