Linear Modeling and Graphical Solution. Exercises

Linear Modeling

1. A family involved in ecological agriculture has a small jam production. This year they have collected 1000kg of apples, 600 kg of plums and 800 kg of peach. The cost generated by each kg of fruit collected is the following: apple 0.40 euro/kg, plum 0.60 euro/kg and peach 0.80 euro/kg.

   By using the fruit collected, they produce these kinds of jam:

   - One flavour jam: apple-jam, plum-jam, peach-jam.
   - Two flavours jam: apple and plum jam, apple and peach jam.

   1kg of jam is obtained from each kg of fruit. Fruit is mixed fifty-fifty in the two flavours jams.

   The jam is sold to a delicatessen shop, at the price of 2 euros each one flavour jam kg, and at the price of 2.5 euros each two flavours jam kg. The shop demands to receive the following minimum quantity of one flavour jam: 175 kg apple-jam, 160 kg plum-jam and 150 kg peach-jam.

   The shop does not demand a minimum amount of two flavours jam, but is ready to buy any quantity offered by the family. Formulate a linear model that can be used to maximize the benefit obtained by the family from the jam production.

2. A food company produces two kinds of fruit salads: normal and low calorie. The characteristics of the fruits used and their prices are shown in the following table:

Fruit	Calories c/kg	Calcium mg/kg	Phosphorus mg/kg	C Vitamin mg/kg	Price euro/kg
1. Cherry	700	250	200	120	7
2. Watermelon	300	100	90	60	0.9
3. Mango	580	150	220	50	4
4. Orange	490	400	200	550	1.6
5. Melon	300	140	160	300	1.4
6. Banana	900	90	280	100	1.5

   Each kg of fruit salad has to hold the following requirements:

   - Normal fruit salad: at least 150 mg of calcium, at least 200 mg of phosphorus and at least 200 mg C vitamin.
   - Low calorie fruit salad: no more than 400 calories, at least 100 mg phosphorus and at least 250 mg C vitamin.

In order to make them tempting, fruits of different colours will be mixed like this:

- Red-coloured fruits: cherry, watermelon, at least 10%.
- Orange-coloured fruits: mango, orange, at least 30%.
- White-coloured fruits: melon, banana, at least 20%.

The minimum cost composition must be found for the two kinds of fruit salads. Formulate a linear model to represent the problem.

3. Children living in a city intend to enjoy their summer in one of the two summer camps near the city. Summer camp S_1 is 8 km away and summer camp S_2 26 km away. It has to be guaranteed that all children will be accepted either in one summer camp or in the other. If possible, they all prefer to go to the nearest one.

   Children whose mother tongue is Basque are majority among all the children, as it can be seen in the table:

Mother tongue	Children
Basque	650 girls, 600 boys
Spanish	475 girls, 475 boys

   It must be guaranteed that the majority of the children going to each summer camp will be girls and that the mother tongue of the majority of them will be Basque. In summer camp S_1 there is no place for more than 800 children. Formulate a linear model to solve the problem.

4. A textile enterprise wants to organize the production for the next six months. Two types of suits are produced: suits made of new fabric and suits made of recycled fabric. Demands for the following six months are the following:

Month	Suits made of new fabric	Suits made of recycled fabric
1	100	100
2	300	150
3	500	300
4	600	200
5	200	100
6	450	300

   Each suit made of new fabric costs 70 euros and each suit made of recycled fabric costs 60 euros. Suits that are not sold can be stored in the warehouse, and will be used to satisfy

the demand of next months. The cost of storing a suit is of 1 euro per month. The storage capacity is unlimited. However, the monthly production capacity is limited to 400 new fabric suits and 200 recycled fabric suits.

The objective of the enterprise is to satisfy the demand for the next months with the minimum cost. Formulate a linear model to solve the problem.

5. Among the 20 children in a classroom, a small group must be selected to take part in an inter-school competition in a television program. The group must consist of at least 3 children. The smaller the group, the higher priority they have to take part in the competition.

   The competition consists in answering questions about six different topics. Children have been examined at school, and the selection will be based on the marks they obtained. The following table shows the marks obtained by the 20 children:

Topic	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	7	8	6	5	7	9	10	5	5	7	6	8	6	7	7	6	5	7	9	5
2	7	6	5	5	9	7	7	7	6	7	6	7	8	8	6	5	6	7	7	7
3	7	7	7	6	6	5	6	7	9	7	5	5	7	7	10	6	7	8	5	7
4	8	7	6	9	7	7	5	7	8	9	6	6	7	5	7	7	7	5	6	10
5	6	5	7	6	6	5	7	7	7	6	7	5	9	7	7	7	10	6	5	5
6	7	7	10	7	5	6	6	9	6	7	8	9	5	7	7	10	6	7	7	5

   The aim is to select the minimum number of children to form the group, but making sure that, for each one of the topics, there is at least a child in the group that is good enough (one who obtained a mark of 8 or higher in the exam). Which children will be selected to form the group? How many of them will be selected? Formulate a linear model that will enable the solving of the problem.

Graphical Solution

Solve graphically the following linear models, and determine the type of solution you obtain.

1. $\min z = 2x_1 - x_2$

   subject to

   $$\begin{aligned} 6x_1 - 6x_2 &\leq 3 \\ x_1 + x_2 &\leq 2 \\ x_1 + 2x_2 &\geq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2. $\max z = -6x_1 - 2x_2$

   subject to

   $$\begin{aligned} x_1 - 4x_2 &\leq 4 \\ x_1 + x_2 &\geq 4 \\ 8x_1 - 4x_2 &\geq -8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

3. $\max z = 2x_1 + 4x_2$

subject to

$$\begin{aligned} 3x_1 - 3x_2 &\leq 6 \\ x_1 + 2x_2 &\leq 4 \\ 2x_1 + x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

4. $\max z = x_1 + 4x_2$

subject to

$$\begin{aligned} x_1 - 2x_2 &\leq 1 \\ 2x_1 + 2x_2 &\geq 1 \\ 3x_1 - x_2 &\geq -3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

5. $\min z = x_1 + 2x_2$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 6 \\ x_1 + 2x_2 &\geq 2 \\ x_1 - x_2 &\leq 2 \\ 6x_1 - 4x_2 &\geq -4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

6. $\max z = x_1 + 2x_2$

subject to

$$\begin{aligned} 3x_1 - 2x_2 &\leq -2 \\ x_1 + x_2 &\leq 4 \\ 2x_1 + x_2 &\geq 2 \\ x_1 - x_2 &\geq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

7. $\max z = 6x_1 + 4x_2$

subject to

$$\begin{aligned} 2x_1 - 2x_2 &\geq -1 \\ x_1 + 2x_2 &\leq 2 \\ 3x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

8. $\min z = -x_1 - 3x_2$

subject to

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ 6x_1 - 3x_2 &\geq -6 \\ x_1 - 2x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$