Dualtasuna. Soluzioak

1. Eredu linealen dualen kalkulua.

1.1.
$$\begin{aligned} \max\ G = y_1 + 7y_2 + 10y_3 \\ \text{hauen mende} \\ y_1 + 2y_2 + y_3 &\geq 2 \\ 2y_1 - 2y_2 + 2y_3 &\leq 3 \\ 5y_1 + 4y_2 + y_3 &= -4 \\ y_1, y_3 \geq 0, y_2 &: \text{ez-murriztua} \end{aligned}$$

1.2.
$$\begin{aligned} \max\ G = -7y_1 + 12y_2 + 5y_3 \\ \text{hauen mende} \\ 4y_1 + 2y_2 + 2y_3 &\leq 1 \\ -y_1 - 4y_2 + 8y_3 &\leq 3 \\ 2y_1 + 4y_3 &\leq 1 \\ y_1 \leq 0, y_2, y_3 &\geq 0 \end{aligned}$$

1.3.
$$\begin{aligned} \min\ G = 12y_1 - 8y_2 + 10y_3 \\ \text{hauen mende} \\ 2y_1 - y_2 + 3y_3 &\leq 2 \\ y_1 + 5y_2 + 4y_3 &\geq 2 \\ 2y_1 - 2y_2 - 6y_3 &\geq 5 \\ y_1 : \text{ez-murriztua}, y_2 \leq 0, y_3 &\geq 0 \end{aligned}$$

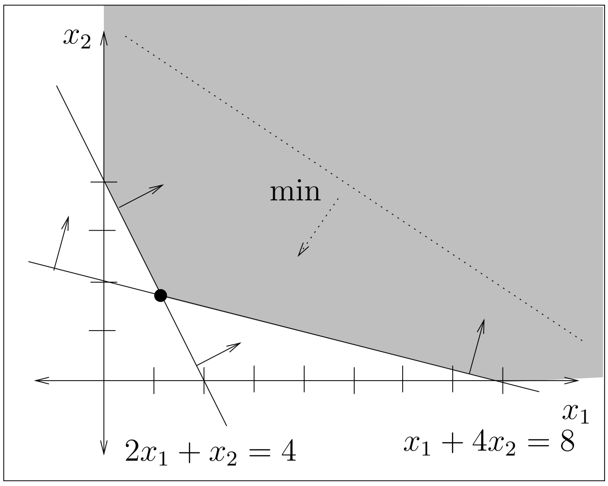
1.4.
$$\begin{aligned} \min\ G = -4y_1 + 2y_2 + 6y_3 \\ \text{hauen mende} \\ y_1 - y_2 + 4y_3 &\geq 1 \\ y_1 + 6y_2 - y_3 &\geq 1 \\ 2y_1 + 2y_2 + y_3 &= 5 \\ y_1 \geq 0, y_2 \leq 0, y_3 &: \text{ez-murriztua} \end{aligned}$$

1.5.
$$\begin{aligned} \max\ G = -6y_1 + 6y_2 + 10y_3 \\ \text{hauen mende} \\ 4y_1 + y_2 + 5y_3 &\geq 4 \\ -2y_1 + y_2 + 2y_3 &\geq 1 \\ 3y_1 + y_2 - y_3 &\leq -1 \\ y_1 + y_2 - y_3 &\leq 2 \\ y_1 \leq 0, y_2 : \text{ez-murriztua}, y_3 &\geq 0 \end{aligned}$$

1.6.
$$\begin{aligned} \min\ G = 14y_1 - 6y_2 + 10y_3 + 3y_4 \\ \text{hauen mende} \\ 2y_1 - y_2 + 4y_3 + y_4 &\geq 1 \\ -4y_1 + 8y_2 + 6y_3 + 9y_4 &\leq 4 \\ y_1, y_3 \geq 0, y_2 \leq 0, y_4 &: \text{ez-murriztua} \end{aligned}$$

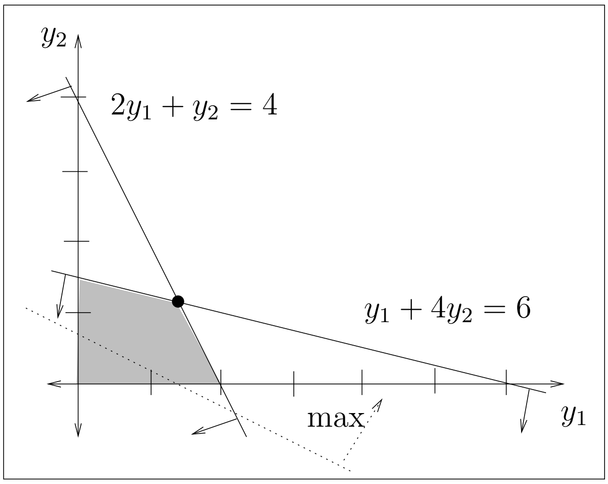
2. Emandako eredu linealaren eta bere dualaren ebazpen grafikoa.

   2.1 Soluzio optimo bakarra, $x_1^* = \frac{8}{7},\ x_2^* = \frac{12}{7},\ z^* = \frac{104}{7}$.

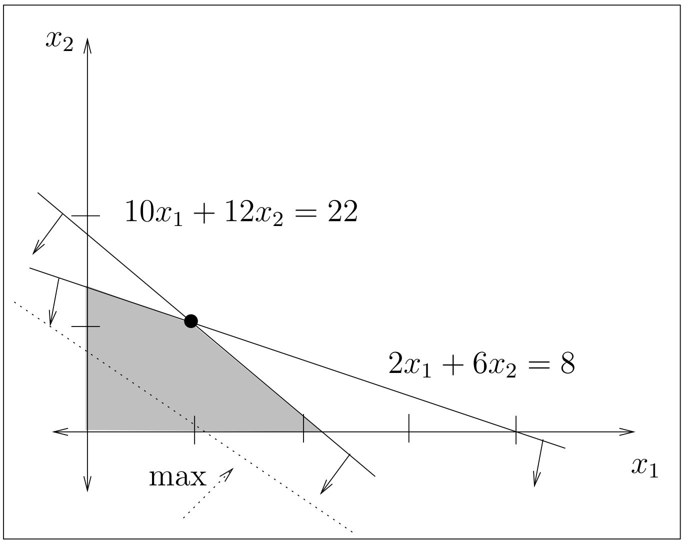


$$
\begin{aligned}
&\max\ G = 4y_1 + 8y_2 \\
&\text{hauen mende} \\
&\qquad 2y_1 + y_2 \leq 4 \\
&\qquad y_1 + 4y_2 \leq 6 \\
&\qquad y_1, y_2 \geq 0
\end{aligned}
$$

Dualak soluzio optimo bakarra du, $y_1^* = \frac{10}{7},\ y_2^* = \frac{8}{7},\ G^* = \frac{104}{7}$.

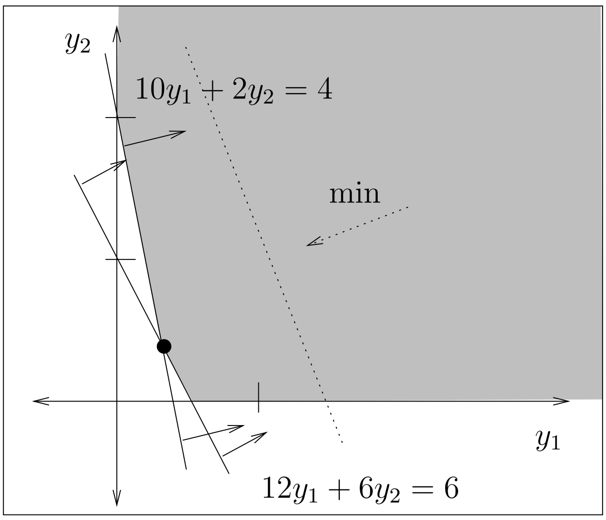


2.2 Soluzio optimo bakarra, $x_1^* = 1,\ x_2^* = 1,\ z^* = 10.$



$$\begin{aligned} \min\ & G = 22y_1 + 8y_2 \\ \text{hauen mende} & \\ & 10y_1 + 2y_2 \geq 4 \\ & 12y_1 + 6y_2 \geq 6 \\ & y_1, y_2 \geq 0 \end{aligned}$$

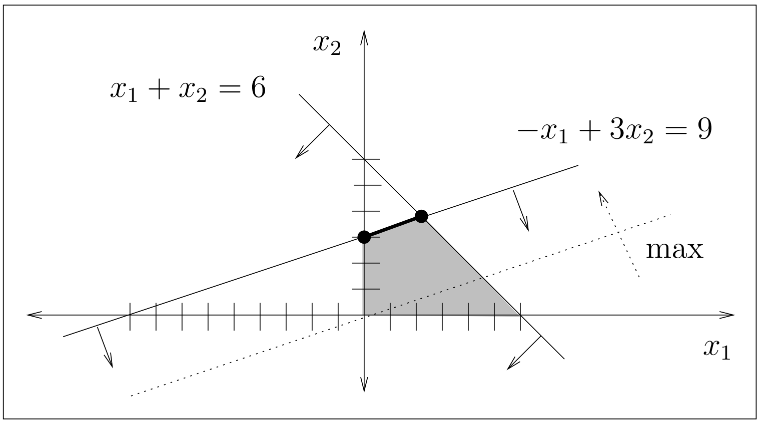
Dualak soluzio optimo bakarra du, $y_1^* = \frac{1}{3},\ y_2^* = \frac{1}{3},\ G^* = 10.$



2.3 Soluzio optimo anizkoitza; bi mutur-puntuak

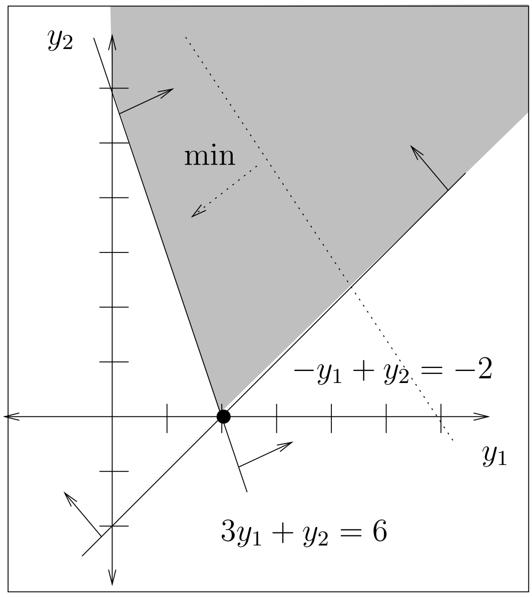
$$x_1^* = 0,\ x_2^* = 3 \quad \text{eta} \quad x_1^* = \frac{9}{4},\ x_2^* = \frac{15}{4},$$

eta bien arteko segmentuko infinitu puntuak. Guztietarako $z^* = 18$ da.

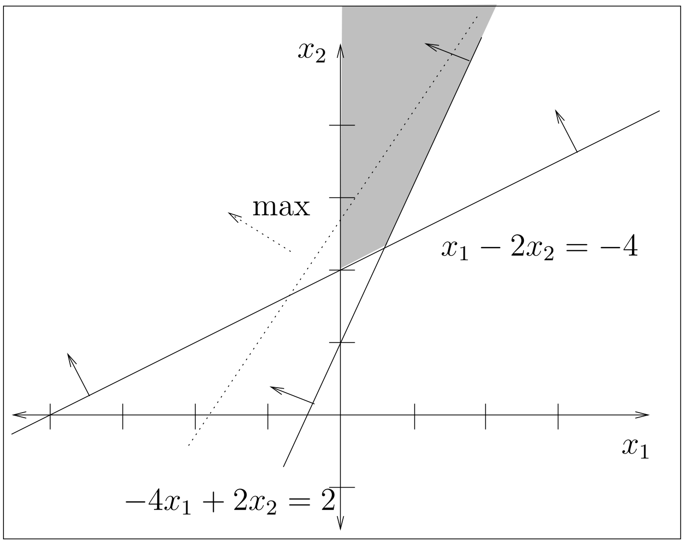


$$\begin{aligned} \min\ & G = 9y_1 + 6y_2 \\ \text{hauen mende} & \\ & -y_1 + y_2 \geq -2 \\ & 3y_1 + y_2 \geq 6 \\ & y_1, y_2 \geq 0 \end{aligned}$$

Dualak soluzio optimo bakarra du, $y_1^* = 2,\ y_2^* = 0,\ G^* = 18.$

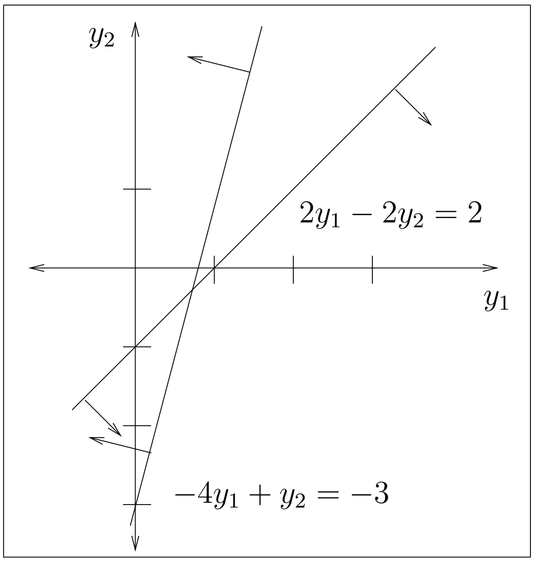


2.4 Soluzio bornegabea da.



$$\begin{aligned} \min\ G &= 2y_1 - 4y_2 \\ \text{hauen mende} & \\ -4y_1 + y_2 &\geq -3 \\ 2y_1 - 2y_2 &\geq 2 \\ y_1 \leq 0, y_2 &\geq 0 \end{aligned}$$

Duala bideraezina da.



3. Eredu linealen ebazpena simplex dual algoritmoa erabiliz.

3.1 Soluzio optimo bakarra.

$$x_1^* = 4, \quad x_2^* = 0, \quad x_3^* = 0, \quad z^* = -8$$

3.2 Soluzio optimo bakarra.

$$x_1^* = 0, \quad x_2^* = 3, \quad x_3^* = 8, \quad x_4^* = 0, \quad z^* = 27$$

3.3 Soluzio optimo anizkoitza, $z^* = -30$.

3.4 Problema bideraezina.

3.5 Problema bideraezina.

3.6 Soluzio optimo bakarra.

$$x_1^* = 0, \quad x_2^* = 6, \quad x_3^* = 0, \quad x_4^* = 12, \quad z^* = 84$$

3.7 Soluzio optimo bakarra.

$$x_1^* = 11, \quad x_2^* = 0, \quad x_3^* = 1, \quad x_4^* = 0, \quad z^* = 35$$

3.8 Soluzio bornegabea.

3.9 Soluzio bornegabea.

4. 4.1 Eredu duala:

$$\begin{aligned} \min\ G &= 2y_1 + 3y_2 + 3y_3 + 2y_4 \\ \text{hauen mende} & \\ y_1 + 2y_2 + 2y_3 + 4y_4 &\geq 10 \\ 2y_1 + y_2 + 2y_3 + y_4 &\geq 6 \\ y_1, y_2, y_3, y_4 &\geq 0 \end{aligned}$$

4.2 Eredu dualaren soluzio optimoa simplex dual algoritmoa erabiliz:

$$y_1^* = 2, \quad y_2^* = 0, \quad y_3^* = 0, \quad y_4^* = 2, \quad G^* = 8.$$

4.3 Dualaren taula optimotik lortutako primalaren soluzio optimoa:

$$x_1^* = \frac{2}{7}, \quad x_2^* = \frac{6}{7}, \quad z^* = 8.$$

5. 5.1 Eredu duala:

$$\begin{aligned} \max\ G &= 20y_1 + 16y_2 + 18y_3 + 21y_4 \\ \text{hauen mende} & \\ 4y_1 + 6y_2 + 4y_3 + 4y_4 &\leq 30 \\ 2y_1 + 4y_2 + 2y_3 + 4y_4 &\leq 28 \\ y_1, y_2, y_3, y_4 &\geq 0 \end{aligned}$$

5.2 Eredu dualaren soluzio optimoa simplex primal algoritmoa erabiliz:

$$y_1^* = 1, \quad y_2^* = 0, \quad y_3^* = 0, \quad y_4^* = \frac{13}{2}, \quad G^* = \frac{313}{2}.$$

5.3 Dualaren taula optimotik lortutako primalaren soluzio optimoa:

$$x_1^* = \frac{19}{4}, \quad x_2^* = \frac{1}{2}, \quad z^* = \frac{313}{2}.$$

6. 6.1 (a) Ereduaren soluzio optimoa:

$$x_1^* = \frac{7}{2}, \quad x_2^* = \frac{3}{2}, \quad x_3^* = 0 \quad z^* = \frac{57}{2}.$$

(b) Dualaren soluzio optimoa $y_1^* = \frac{3}{20}$, $y_2^* = \frac{1}{4}$, $G^* = \frac{57}{2}$ da. Eredu duala hau da:

$$\begin{aligned} \min\ G = 90y_1 + 60y_2 \\ \text{hauen mende} \\ 15y_1 + 15y_2 \geq 6 \\ 25y_1 + 5y_2 \geq 5 \\ 30y_1 + 15y_2 \geq 4 \\ y_1, y_2 \geq 0 \end{aligned}$$

(c) Itzal-prezioen interpretazioa b_1 baliabiderako.

$$\overset{\wedge}{\mathbf{b}} = \begin{pmatrix} 91 \\ 60 \end{pmatrix}, \quad \mathbf{x}_B = \mathbf{B}^{-1}\overset{\wedge}{\mathbf{b}} = \begin{pmatrix} \frac{31}{20} \\ \frac{209}{60} \end{pmatrix} \geq \mathbf{0}$$

b_1 baliabidearen itzal-prezioa y_1^* da. b_1 baliabidea 90 unitatetik 91 unitatera pasatzen bada, $\hat{z} = z + y_1^* = \frac{57}{2} + \frac{3}{20} = \frac{573}{20}$ izango da, eta soluzio optimo berria:

$$x_1^* = \frac{209}{60}, \quad x_2^* = \frac{31}{20}, \quad x_3^* = 0$$

Itzal-prezioen interpretazioa b_2 baliabiderako.

$$\overset{\wedge}{\mathbf{b}} = \begin{pmatrix} 90 \\ 61 \end{pmatrix}, \quad \mathbf{x}_B = \mathbf{B}^{-1}\overset{\wedge}{\mathbf{b}} = \begin{pmatrix} \frac{29}{20} \\ \frac{215}{60} \end{pmatrix} \geq \mathbf{0}.$$

b_2 baliabidearen itzal-prezioa y_2^* da. b_2 baliabidea 60 unitatetik 61 unitatera pasatzen bada, $\hat{z} = z + y_2^* = \frac{57}{2} + \frac{1}{4} = \frac{115}{4}$ izango da, eta soluzio optimo berria:

$$x_1^* = \frac{215}{60}, \quad x_2^* = \frac{29}{20}, \quad x_3^* = 0.$$

6.2 (a) Ereduaren soluzio optimoa.

$$x_1^* = 12, \quad x_2^* = 0, \quad x_3^* = 0 \quad z^* = 24.$$

(b) Dualaren soluzio optimoa $y_1^* = 2, \;\; y_2^* = 0, \;\; G^* = 24$ da. Eredu duala hau da:

$$\begin{aligned} \min\ G = 12y_1 + 8y_2 \\ \text{hauen mende} \qquad\qquad \\ y_1 + 4y_2 \geq 2 \\ 2y_1 + 2y_2 \geq 1 \\ 4y_1 \geq -1 \\ y_1 \geq 0, y_2 \leq 0 \end{aligned}$$

(c) Itzal-prezioen interpretazioa b_1 baliabiderako.

$$\overset{\wedge}{\mathbf{b}} = \begin{pmatrix} 13 \\ 8 \end{pmatrix}, \quad \mathbf{x}_B = \mathbf{B}^{-1}\overset{\wedge}{\mathbf{b}} = \begin{pmatrix} 44 \\ 13 \end{pmatrix} \geq \mathbf{0}.$$

b_1 baliabidearen itzal-prezioa y_1^* da. b_1 baliabidea 12 unitatetik 13 unitatera pasatzen bada, $\hat{z} = z + y_1^* = 24 + 2 = 26$ izango da, eta soluzio optimo berria:

$$x_1^* = 13, \quad x_2^* = 0, \quad x_3^* = 0.$$

Itzal-prezioen interpretazioa b_2 baliabiderako.

$$\overset{\wedge}{\mathbf{b}} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}, \quad \mathbf{x}_B = \mathbf{B}^{-1}\overset{\wedge}{\mathbf{b}} = \begin{pmatrix} 41 \\ 12 \end{pmatrix} \geq \mathbf{0}.$$

b_2 baliabidearen itzal-prezioa y_2^* da. b_2 baliabidea 8 unitatetik 7 unitatera pasatzen bada, $\hat{z} = z - y_2^* = 24 - 0 = 24$ izango da, eta soluzio optimo berria:

$$x_1^* = 12, \quad x_2^* = 0, \quad x_3^* = 0.$$