# Simplex Metodoa. Soluzioak

1. Ereduen forma estandarrak ondokoak dira:

1.1

$$\begin{aligned} \max z &= 2x_1 + 4x_2 - 4x_3' + 4x_3'' + 0x_4 + 0x_5 \\ &\text{hauen mende} \\ 3x_1 + 2x_2 + 4x_3' - 4x_3'' - x_4 &= 1 \\ 4x_1 - 3x_2 &= 2 \\ 2x_1 + x_2 + 6x_3' - 6x_3'' + x_5 &= 3 \\ x_1, x_2, x_3', x_3'', x_4, x_5 \geq 0, x_3 &= x_3' - x_3'' \end{aligned}$$

1.2

$$\begin{aligned} \max (-z) &= -2x_1 + 3x_2 - x_3 + 0x_4 + 0x_5 \\ &\text{hauen mende} \\ x_1 - 5x_2 + 6x_3 - x_4 &= 8 \\ -x_1 + 4x_2 - x_5 &= 12 \\ 2x_1 - x_2 + 4x_3 &= 5 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

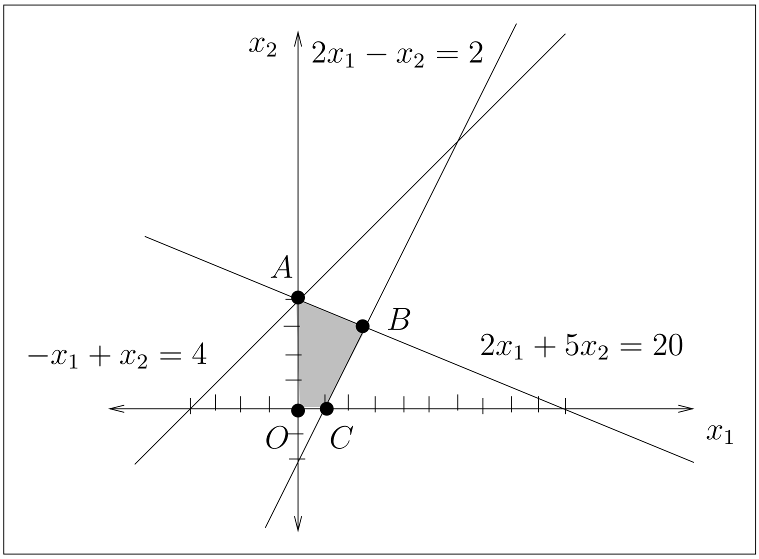
1.3

$$\begin{aligned} \max (-z) &= 2x_1' - 2x_2 + 4x_3 + 0x_4 + 0x_5 \\ &\text{hauen mende} \\ -2x_1' + 2x_2 + 2x_3 &= 10 \\ -2x_1' - 6x_2 + x_3 - x_4 &= 10 \\ x_1' + 3x_2 - x_5 &= 3 \\ x_1', x_2, x_3, x_4, x_5 \geq 0, x_1' &= -x_1 \end{aligned}$$

1.4

$$\begin{aligned} \max z &= -3x_1' - 7x_2 + 5x_3' - 5x_3'' + 0x_4 + 0x_5 \\ &\text{hauen mende} \\ -x_2 + x_3' - x_3'' - x_4 &= 9 \\ x_1' - 2x_3' + 2x_3'' - x_5 &= 5 \\ -4x_1' - x_2 &= 6 \\ x_1', x_2, x_3', x_3'', x_4, x_5 &\geq 0 \\ x_1' = -x_1, x_3 &= x_3' - x_3'' \end{aligned}$$

2. Oinarriko soluzioen kalkulua eta puntuen kokapena grafikoan.




10 oinarriko soluzio daude.

- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3), \mathbf{x}_B = \begin{pmatrix} \frac{5}{2} \\ 3 \\ \frac{7}{2} \end{pmatrix} \geq \mathbf{0}, \quad x_1 = \frac{5}{2}, x_2 = 3 \quad \rightarrow \quad B$

- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_4), \mathbf{x}_B = \begin{pmatrix} 6 \\ 10 \\ -42 \end{pmatrix} \ngeq \mathbf{0}, \quad x_1 = 6, x_2 = 10$

- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_5), \mathbf{x}_B = \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} \geq \mathbf{0}, \quad x_1 = 0, x_2 = 4 \quad \rightarrow \quad A$ (endekatua)

- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_3\ \mathbf{a}_4), \mathbf{x}_B = \begin{pmatrix} 1 \\ 5 \\ 18 \end{pmatrix} \geq \mathbf{0}, \quad x_1 = 1, x_2 = 0 \quad \rightarrow \quad C$

- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_3\ \mathbf{a}_5), \mathbf{x}_B = \begin{pmatrix} 10 \\ 14 \\ -18 \end{pmatrix} \ngeq \mathbf{0}, \quad x_1 = 10, x_2 = 0$

- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_4\ \mathbf{a}_5), \mathbf{x}_B = \begin{pmatrix} -4 \\ 28 \\ 10 \end{pmatrix} \not\geq \mathbf{0}, \quad x_1 = -4, x_2 = 0$
- $\mathbf{B} = (\mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_4), \mathbf{x}_B = \begin{pmatrix} -2 \\ 6 \\ 30 \end{pmatrix} \not\geq \mathbf{0}, \quad x_1 = 0, x_2 = -2$
- $\mathbf{B} = (\mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_5), \mathbf{x}_B = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix} \geq \mathbf{0}, \quad x_1 = 0, x_2 = 4 \quad \rightarrow \quad A$ (endekatua)
- $\mathbf{B} = (\mathbf{a}_2\ \mathbf{a}_4\ \mathbf{a}_5), \mathbf{x}_B = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix} \geq \mathbf{0}, \quad x_1 = 0, x_2 = 4 \quad \rightarrow \quad A$ (endekatua)
- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_4\ \mathbf{a}_5), \mathbf{x}_B = \begin{pmatrix} 4 \\ 20 \\ 2 \end{pmatrix} \geq \mathbf{0}, \quad x_1 = 0, x_2 = 0 \quad \rightarrow \quad O$

3. Soluzio optimoa: $x_1^* = 1, \quad x_2^* = 1, \quad x_3^* = 0, \quad z^* = 7.$

4. $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3)$ oinarriari dagokion oinarriko soluzioa optimoa dela frogatuta geratzen da, $z_4 - c_4 = 1$, $z_5 - c_5 = 2$, $z_6 - c_6 = 1$ positiboak direlako. Soluzio optimoa: $x_1^* = 1$, $x_2^* = 3, \quad x_3^* = 1, \quad z^* = 13.$

5. Taulako koordenatu-bektoreen egiaztapena eta taula osatzea.

   5.1 Erroreak daude $\mathbf{y}_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ bektorean. Ez da betetzen $\mathbf{y}_1 = \mathbf{B}^{-1}\mathbf{a}_1$.

$$\mathbf{y}_1 = \mathbf{B}^{-1}\mathbf{a}_1 = \begin{pmatrix} 1 & -2 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \\ 1 \end{pmatrix}$$

5.2 Ez dago errorerik $\mathbf{y}_5 = \begin{pmatrix} -2 \\ \frac{1}{2} \\ -1 \end{pmatrix}$ bektorean, $\mathbf{y}_5 = \mathbf{B}^{-1}\mathbf{a}_5$ betetzen delako.

$$\mathbf{y}_5 = \mathbf{B}^{-1}\mathbf{a}_5 = \begin{pmatrix} 1 & -2 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ \frac{1}{2} \\ -1 \end{pmatrix}$$

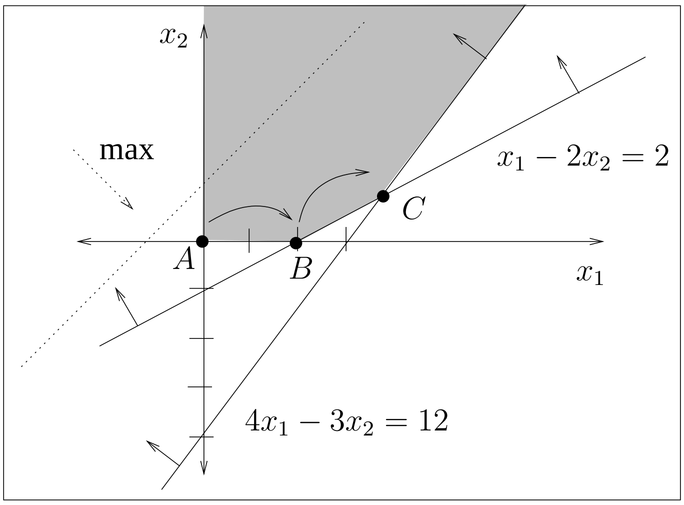
5.3 Taulan falta diren kalkuluak:

$$\mathbf{y}_3 = \begin{pmatrix} 4 \\ \frac{1}{2} \\ 2 \end{pmatrix}, \quad z_3 - c_3 = -1, \quad \mathbf{x}_B = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \quad z = 8.$$

6. Soluzio optimoen kalkulua simplex algoritmoa erabiliz, eta mutur-puntuen kokapena grafikoan.

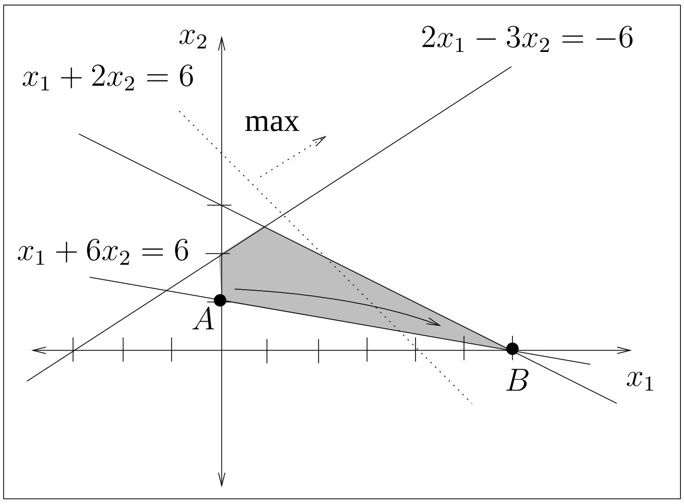
6.1 Soluzio optimo bakarra, $x_1^* = \frac{18}{5}, x_2^* = \frac{4}{5}$.

- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_4), \quad \mathbf{x}_B = \begin{pmatrix} 2 \\ 12 \end{pmatrix}, \quad x_1 = 0, x_2 = 0 \quad \rightarrow \quad A$
- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_4), \quad \mathbf{x}_B = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \quad x_1 = 2, x_2 = 0 \quad \rightarrow \quad B$
- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_2), \quad \mathbf{x}_B = \begin{pmatrix} \frac{18}{5} \\ \frac{4}{5} \end{pmatrix}, \quad x_1^* = \frac{18}{5}, x_2^* = \frac{4}{5} \quad \rightarrow \quad C$



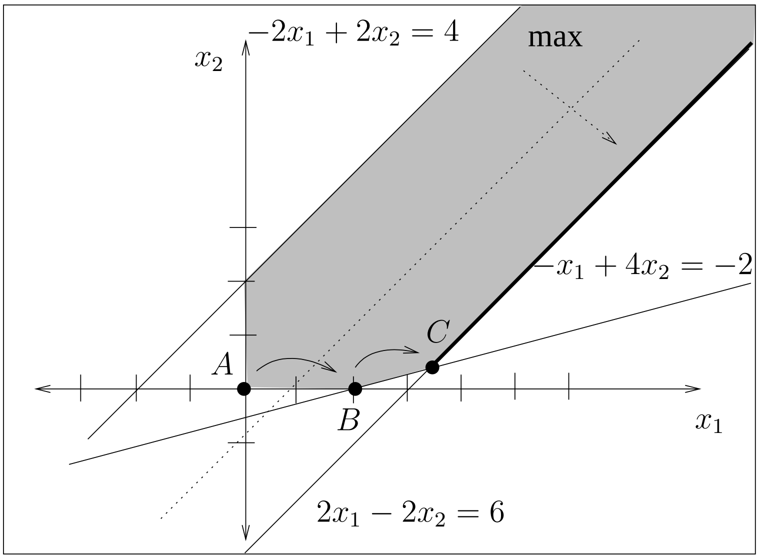
6.2 Soluzio optimo bakarra, endekatua, $x_1^* = 6, x_2^* = 0, z^* = 6$.

- $\mathbf{B} = (\mathbf{a}_{w1}\ \mathbf{a}_4\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$
- $\mathbf{B} = (\mathbf{a}_2\ \mathbf{a}_4\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, \quad x_1 = 0, x_2 = 1 \quad \rightarrow \quad A$
- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_4\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 6 \\ 18 \\ 0 \end{pmatrix}, \quad x_1 = 6, x_2 = 0 \quad \rightarrow \quad B$
- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_4\ \mathbf{a}_3), \quad \mathbf{x}_B = \begin{pmatrix} 6 \\ 18 \\ 0 \end{pmatrix}, \quad x_1^* = 6, x_2^* = 0 \quad \rightarrow \quad B$



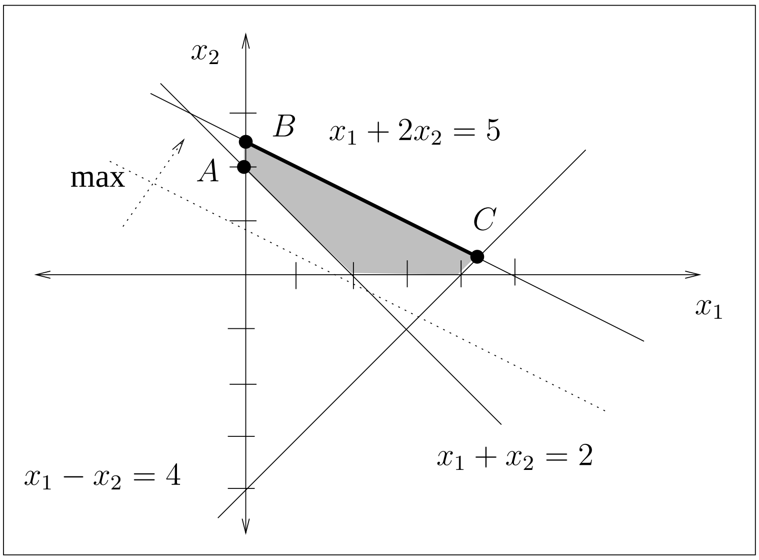
6.3 Soluzio optimo anizkoitza, $z^* = 12$.

- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_4\ \mathbf{a}_5)$, $\quad \mathbf{x}_B = \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix}$, $\quad x_1 = 0, x_2 = 0 \quad \rightarrow \quad A$
- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_4\ \mathbf{a}_1)$, $\quad \mathbf{x}_B = \begin{pmatrix} 8 \\ 2 \\ 2 \end{pmatrix}$, $\quad x_1 = 2, x_2 = 0 \quad \rightarrow \quad B$
- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_2\ \mathbf{a}_1)$, $\quad \mathbf{x}_B = \begin{pmatrix} 10 \\ \frac{1}{3} \\ \frac{10}{3} \end{pmatrix}$, $\quad x_1^* = \frac{10}{3}, x_2^* = \frac{1}{3} \quad \rightarrow \quad C$



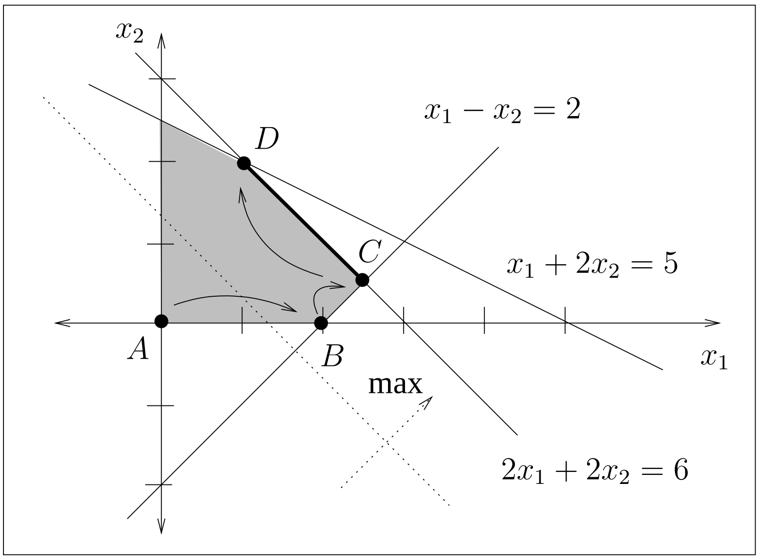
6.4 Soluzio optimo anizkoitza, $z^* = 5$.

- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_{w1}\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$
- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_2\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}, \quad x_1 = 0, x_2 = 2 \quad \rightarrow \quad A$
- $\mathbf{B} = (\mathbf{a}_4\ \mathbf{a}_2\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} \frac{1}{2} \\ \frac{5}{2} \\ \frac{13}{2} \end{pmatrix}, \quad x_1^* = 0, x_2^* = \frac{5}{2} \quad \rightarrow \quad B$
- $\mathbf{B} = (\mathbf{a}_4\ \mathbf{a}_2\ \mathbf{a}_1), \quad \mathbf{x}_B = \begin{pmatrix} \frac{8}{3} \\ \frac{1}{3} \\ \frac{13}{3} \end{pmatrix}, \quad x_1^* = \frac{13}{3}, x_2^* = \frac{1}{3} \quad \rightarrow \quad C$



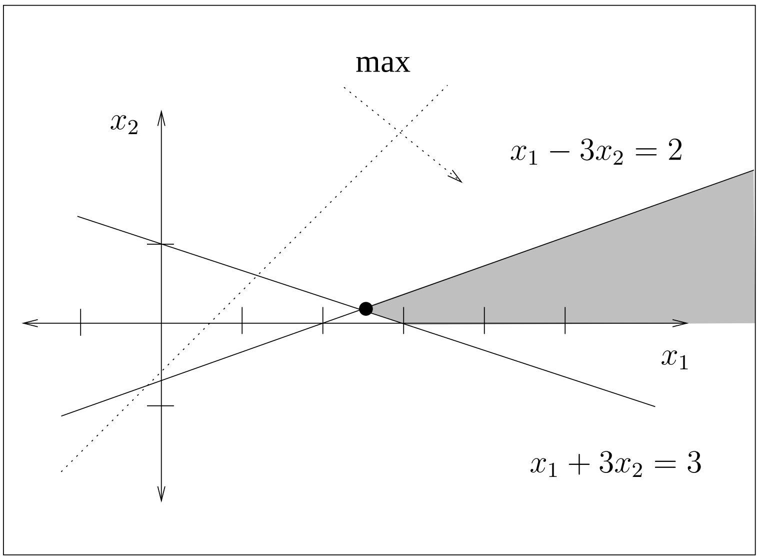
6.5 Soluzio optimo anizkoitza, $z^* = 6$.

- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_4\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}, \quad x_1 = 0, x_2 = 0 \quad \rightarrow \quad A$
- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_4\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}, \quad x_1 = 2, x_2 = 0 \quad \rightarrow \quad B$
- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} \frac{5}{2} \\ \frac{1}{2} \\ \frac{3}{2} \end{pmatrix}, \quad x_1^* = \frac{5}{2}, x_2^* = \frac{1}{2} \quad \rightarrow \quad C$
- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3), \quad \mathbf{x}_B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad x_1^* = 1, x_2^* = 2 \quad \rightarrow \quad D$



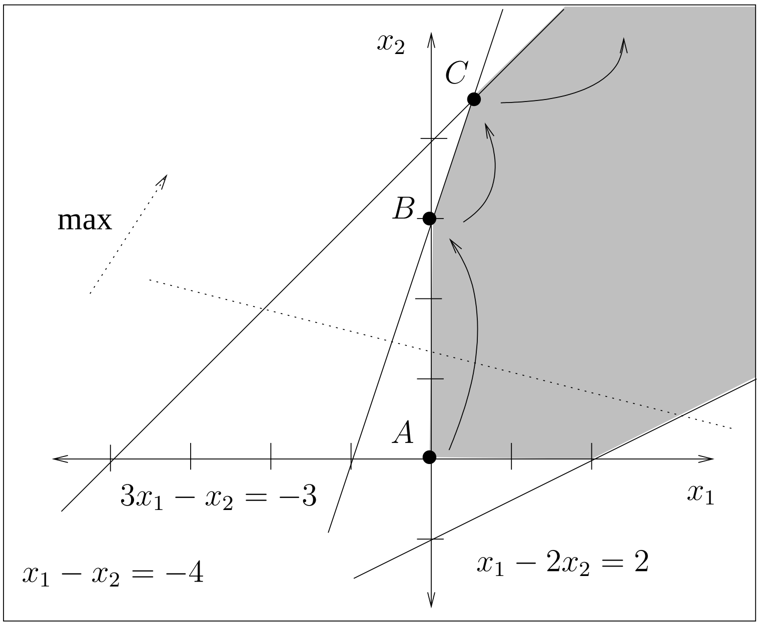
6.6 Soluzio bornegabea.

- $\mathbf{B} = (\mathbf{a}_{w1}\ \mathbf{a}_{w2}), \quad \mathbf{x}_B = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$
- $\mathbf{B} = (\mathbf{a}_{w1}\ \mathbf{a}_{1}), \quad \mathbf{x}_B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
- $\mathbf{B} = (\mathbf{a}_{2}\ \mathbf{a}_{1}), \quad \mathbf{x}_B = \begin{pmatrix} \frac{1}{6} \\ \frac{5}{2} \end{pmatrix}, \quad x_1 = \frac{5}{2}, x_2 = \frac{1}{6}$



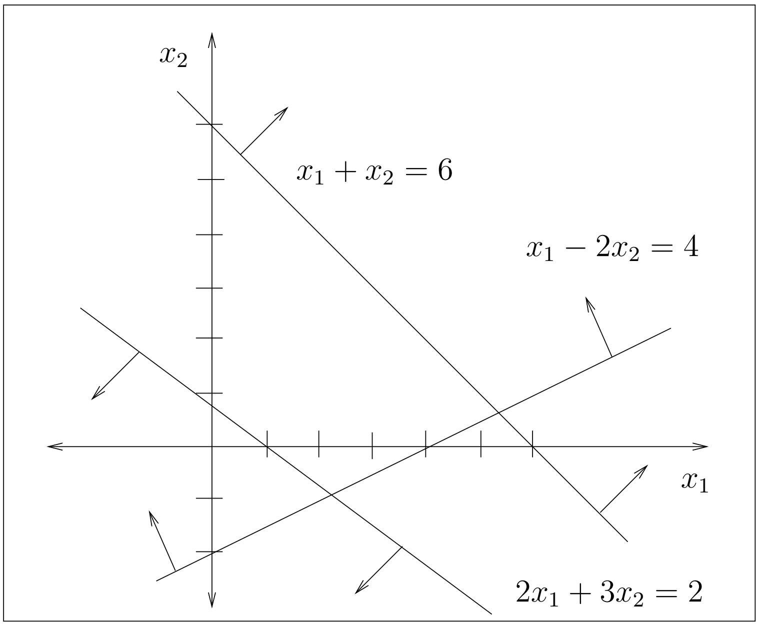
6.7 Soluzio bornegabea.

- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_4\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}, \quad x_1 = 0, x_2 = 0 \quad \rightarrow \quad A$
- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_2\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 1 \\ 3 \\ 8 \end{pmatrix}, \quad x_1 = 0, x_2 = 3 \quad \rightarrow \quad B$
- $\mathbf{B} = (\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} \frac{1}{2} \\ \frac{9}{2} \\ \frac{21}{2} \end{pmatrix}, \quad x_1 = \frac{1}{2}, x_2 = \frac{9}{2} \quad \rightarrow \quad C$



6.8 Problema bideraezina.

- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_{w1}\ \mathbf{a}_5), \quad \mathbf{x}_B = \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix}$
- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_{w1}\ \mathbf{a}_2), \quad \mathbf{x}_B = \begin{pmatrix} \frac{16}{3} \\ \frac{16}{3} \\ \frac{2}{3} \end{pmatrix}$
- $\mathbf{B} = (\mathbf{a}_3\ \mathbf{a}_{w1}\ \mathbf{a}_1), \quad \mathbf{x}_B = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$



7. Soluzio optimoen kalkulua simplex algoritmoa erabiliz.

7.1 Soluzio optimo bakarra, $x_1^* = 0, \quad x_2^* = 8, \quad x_3^* = 0, \quad z^* = 16.$

7.2 Soluzio optimo bakarra, $x_1^* = -1, \quad x_2^* = 7, \quad x_3^* = 0, \quad z^* = -12.$

7.3 Soluzio optimo bakarra, $x_1^* = 5, \quad x_2^* = 6, \quad x_3^* = 0, \quad z^* = -13.$

7.4 Soluzio optimo anizkoitza, $z^* = 24.$

7.5 Soluzio optimo anizkoitza, $z^* = -73.$

7.6 Problema bideraezina.

7.7 Soluzio bornegabea.

7.8 Problema bideraezina.