# 4. SENTIKORTASUNAREN ANALISIA

1. Planteamendu orokorra
2. Adibidea
3. Aldaketak $\mathbf{b}$ bektorean
4. Aldaketak $\mathbf{c}$ bektorean
5. Aldaketak oinarrikoa ez den $\mathbf{a}_j$ batean
6. Aldagai berriak
7. Murrizketa berriak

# 1. Planteamendu orokorra

- **Eredua nasaitze-aldagaiekin**

$$\begin{aligned} \text{max } z &= \mathbf{c}^T\mathbf{x} + \mathbf{0}^T\mathbf{x}_h \\ \text{hauen mende} & \\ \mathbf{A}\mathbf{x} + \mathbf{I}\mathbf{x}_h &= \mathbf{b} \\ \mathbf{x}, \mathbf{x}_h &\geq \mathbf{0} \end{aligned}$$

- **Taula optimoa**

| | Hasierako aldag. $x_1 \; \dots \; x_n$ | Nasaitze-aldag. $x_{n+1} \; \dots \; x_{n+m}$ | |
|---|---|---|---|
| | $\mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{A} - \mathbf{c}^T$ | $\mathbf{c}_B^T\mathbf{B}^{-1}$ | $z = \mathbf{c}_B^T\mathbf{x}_B$ |
| $\mathbf{B}$ | $\mathbf{B}^{-1}\mathbf{A}$ | $\mathbf{B}^{-1}$ | $\mathbf{x}_B = \mathbf{B}^{-1}\mathbf{b}$ |

Taula optimoak bideragarritasun primala ($\mathbf{x}_B \geq \mathbf{0}$) eta bideragarritasun duala ($z_j - c_j \geq 0$) ditu.

Sentikortasunaren analisia taula optimoaren erabilpenean oinarritzen da.

## 2. Adibidea

Ekoizpen-problema bat.

| Baliabideak | Produktuak | | | Baliabideen erabilgarritasuna |
|---|---|---|---|---|
| | $A$ | $B$ | $C$ | |
| 1 | 4 | 2 | 3 | 40 |
| 2 | 2 | 2 | 1 | 30 |
| Irabazia | 3 | 2 | 1 | |

$x_j$ : $A$, $B$ eta $C$ produktu mota bakoitzetik ekoitziko den unitate kopurua, $j = 1, 2, 3$.

$$\max\ z = 3x_1 + 2x_2 + x_3 + 0x_4 + 0x_5$$

hauen mende

$$\begin{aligned} 4x_1 + 2x_2 + 3x_3 + x_4 \phantom{+ x_5} &= 40 \\ 2x_1 + 2x_2 + x_3 \phantom{+ x_4} + x_5 &= 30 \\ x_1, \ldots, x_5 &\geq 0 \end{aligned}$$

# 3. Aldaketak b bektorean

**1. Eredua**

$$\max\ z = \mathbf{c}^T\mathbf{x}$$

hauen mende

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$

$$\mathbf{x} \geq \mathbf{0}$$

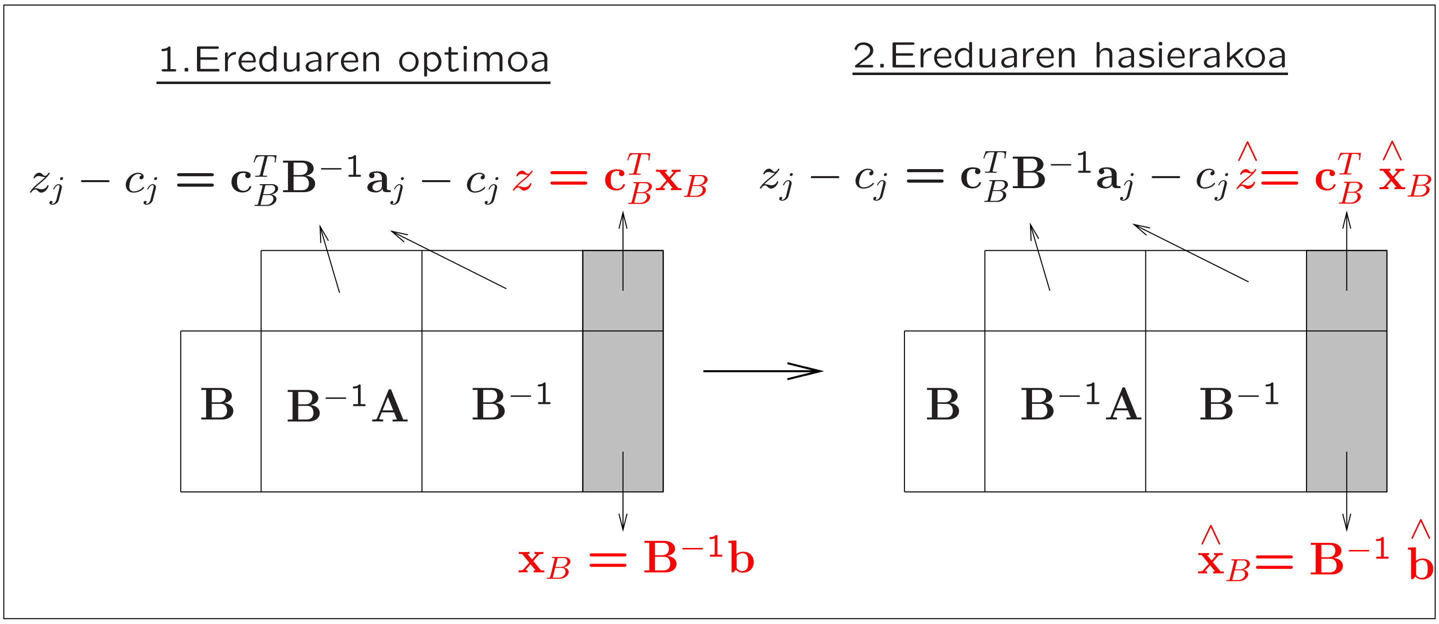
**2. Eredua**

$$\max\ z = \mathbf{c}^T\mathbf{x}$$

hauen mende

$$\mathbf{A}\mathbf{x} \leq \hat{\mathbf{b}}$$

$$\mathbf{x} \geq \mathbf{0}$$




Bi kasu:

1. **kasua.** $\hat{\mathbf{x}}_B \geq \mathbf{0}$ bada, taula optimoa da 2. Eredurako: $\hat{\mathbf{x}}_B$ eta $\hat{z}$ optimoak.

2. **kasua.** $\hat{\mathbf{x}}_B \not\geq \mathbf{0}$ bada, Simplex dual aplikatu.

# 4. Aldaketak $\mathbf{c}$ bektorean

**1. Eredua**

$$\max\ z = \mathbf{c}^T\mathbf{x}$$

hauen mende

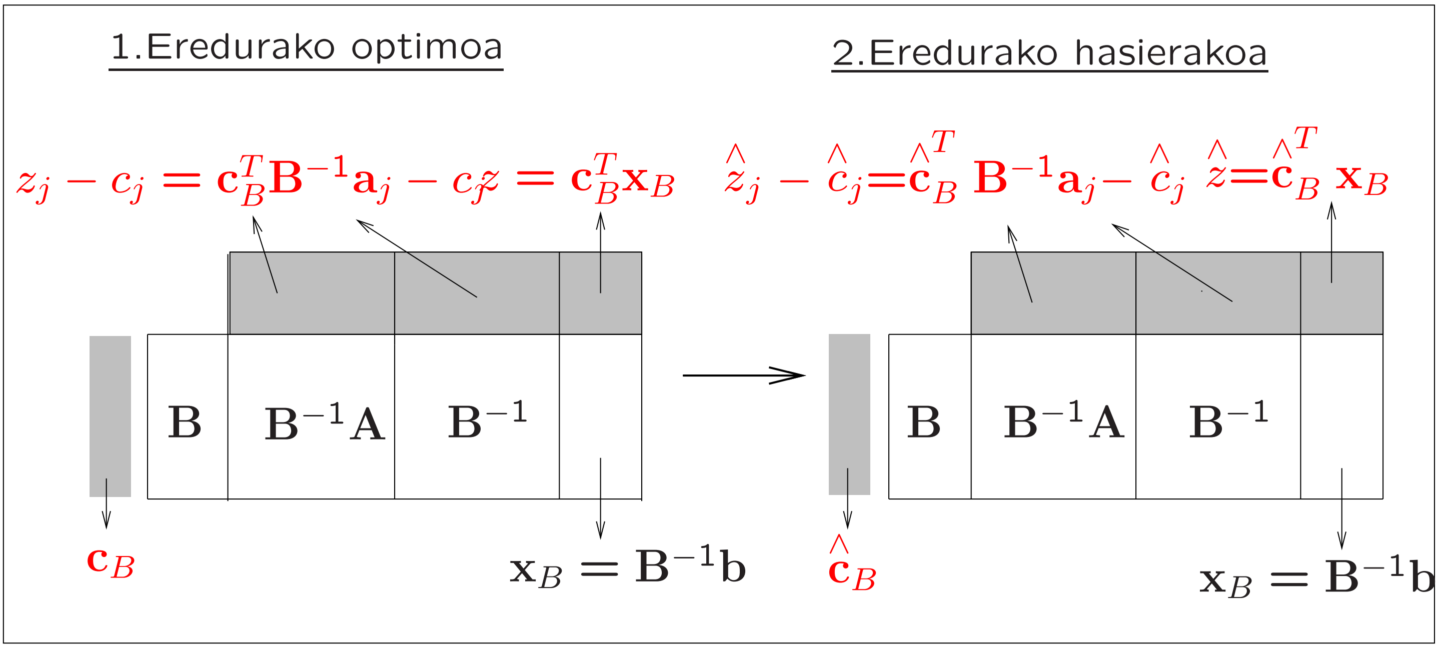
$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$

**2. Eredua**

$$\max\ z = \hat{\mathbf{c}}^T\mathbf{x}$$

hauen mende

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$




Bi kasu:

1. **kasua.** $\hat{z}_j - \hat{c}_j \geq 0$ bada $\forall j$, $\mathbf{x}_B$ eta $\hat{z} = \hat{\mathbf{c}}_B^T\mathbf{x}_B$ optimoak.

2. **kasua.** $\hat{z}_j - \hat{c}_j < 0$ existitzen bada, simplex primal algoritmoa erabili.

# 5. Aldaketak oinarrikoa ez den $\mathbf{a}_j$ batean

**1. Eredua**

$$\max\ z = \mathbf{c}^T\mathbf{x}$$

hauen mende

$$\mathbf{a}_1 x_1 + \cdots + \mathbf{a}_j x_j + \cdots + \mathbf{a}_n x_n \leq \mathbf{b}$$

$$x_1, \cdots, x_n \geq 0$$

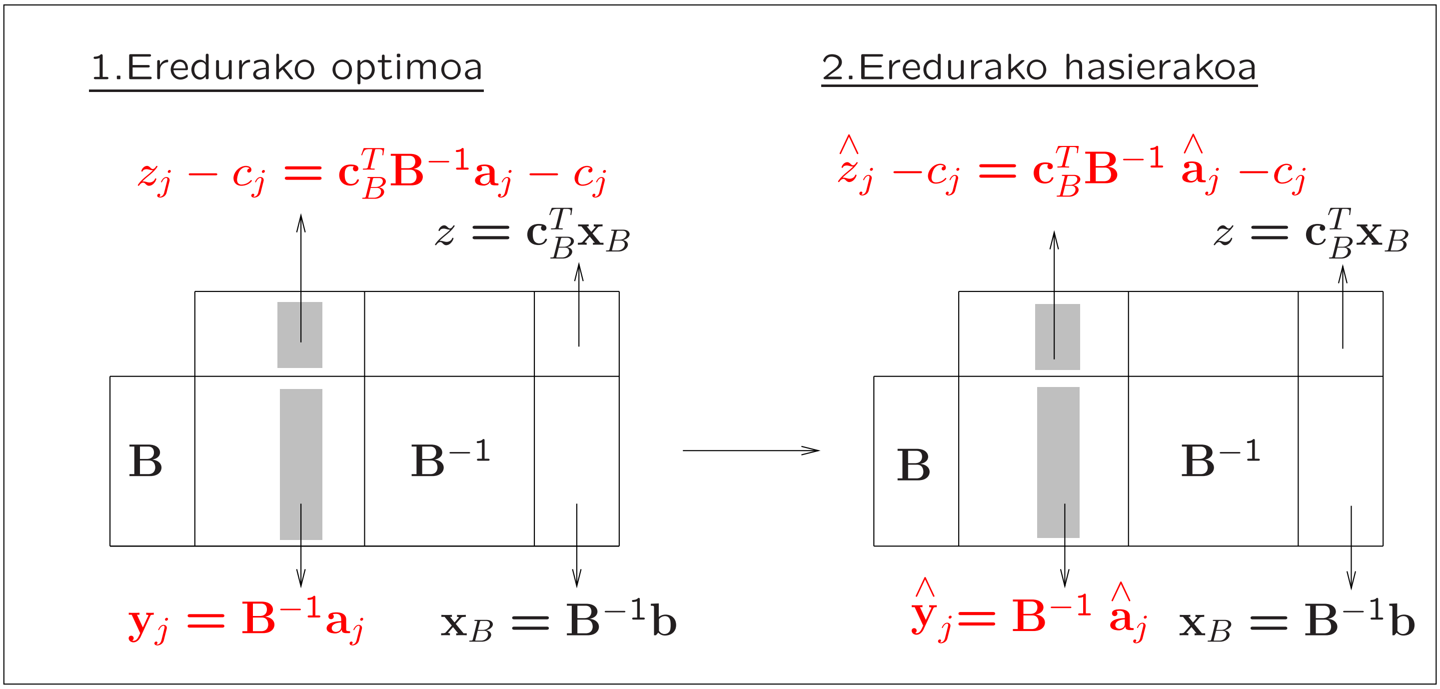
**2. Eredua**

$$\max\ z = \mathbf{c}^T\mathbf{x}$$

hauen mende

$$\mathbf{a}_1 x_1 + \cdots + \hat{\mathbf{a}}_j x_j + \cdots + \mathbf{a}_n x_n \leq \mathbf{b}$$

$$x_1, \cdots, x_n \geq 0$$




Bi kasu:

1. **Kasua.** $\hat{z}_j - c_j \geq 0$ bada, $\mathbf{x}_B$ eta $z$ optimoak.

2. **Kasua** $\hat{z}_j - c_j < 0$ bada, simplex primal algoritmoa erabili.

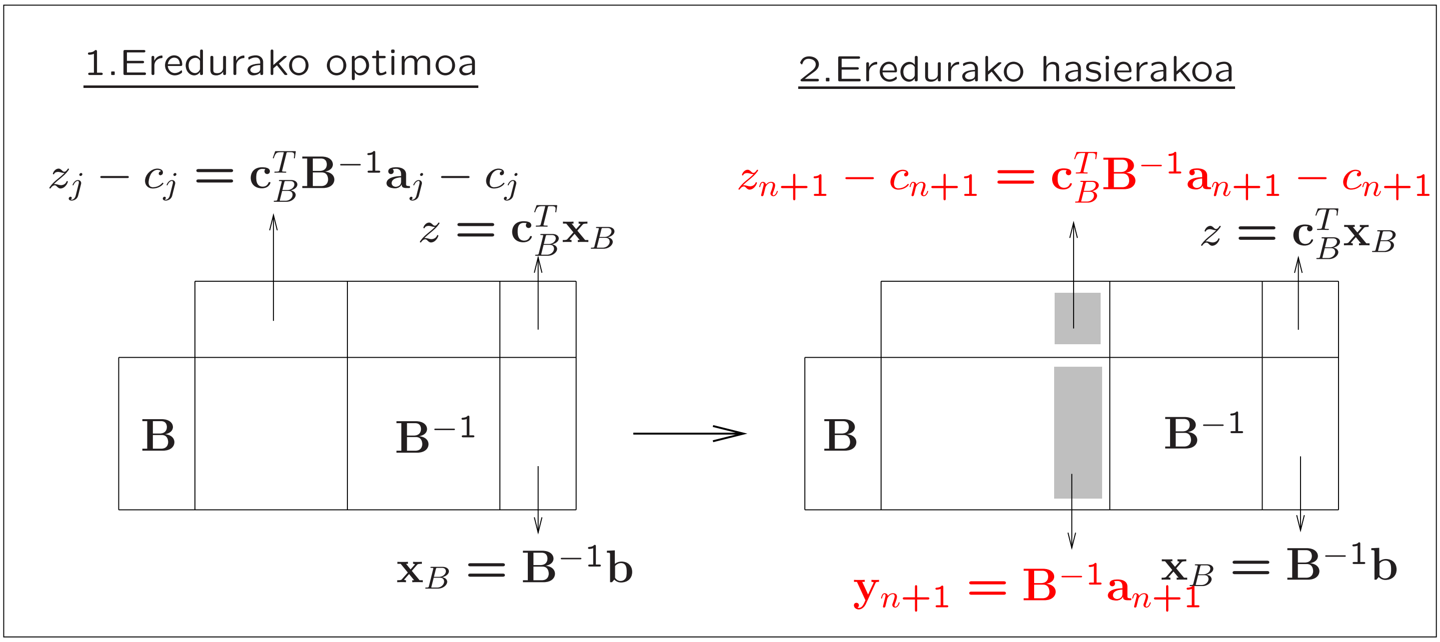
# 6. Aldagai berriak

**1. Eredua**

$$\begin{aligned} \max\ z &= c_1x_1 + \cdots + c_nx_n \\ \text{hauen mende} & \\ \mathbf{a}_1x_1 + \cdots + \mathbf{a}_nx_n &\leq \mathbf{b} \\ x_1, \cdots, x_n &\geq 0 \end{aligned}$$

**2. Eredua**

$$\begin{aligned} \max\ z &= c_1x_1 + \cdots + c_nx_n + c_{n+1}x_{n+1} \\ \text{hauen mende} & \\ \mathbf{a}_1x_1 + \cdots + \mathbf{a}_nx_n + \mathbf{a}_{n+1}x_{n+1} &\leq \mathbf{b} \\ x_1, \cdots, x_n, x_{n+1} &\geq 0 \end{aligned}$$




Bi kasu:

1. **Kasua.** $z_{n+1} - c_{n+1} \geq 0$ bada, $\mathbf{x}_B$ eta $z$ optimoak.

2. **Kasua.** $z_{n+1} - c_{n+1} < 0$ bada, simplex primal algoritmoa erabili.

# 7. Murrizketa berriak

**1. Eredua**

$$\max\ z = c_1x_1 + \cdots + c_nx_n$$

hauen mende

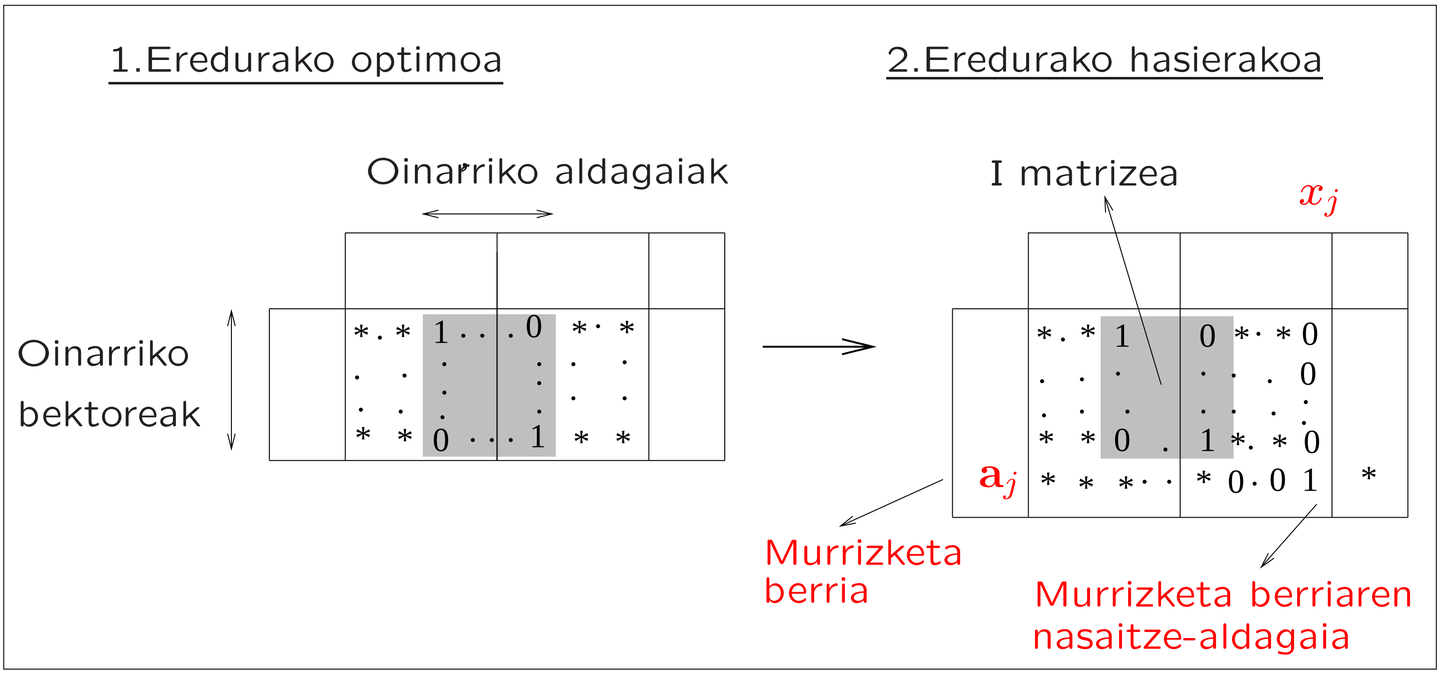
$$\begin{aligned} a_{11}x_1 + \cdots + a_{1n}x_n &\leq b_1 \\ \vdots \qquad\qquad & \quad \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n &\leq b_m \\ x_1, \cdots, x_n &\geq 0 \end{aligned}$$

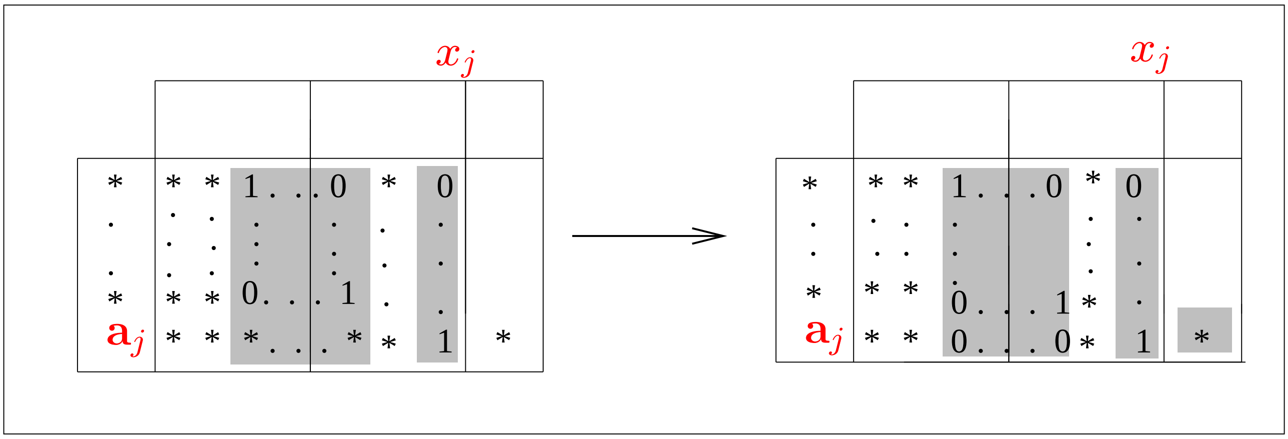
**2. Eredua**

$$\max\ z = c_1x_1 + \cdots + c_nx_n$$

hauen mende

$$\begin{aligned} a_{11}x_1 + \cdots + a_{1n}x_n &\leq b_1 \\ \vdots \qquad\qquad & \quad \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n &\leq b_m \\ a_{m+1,1}x_1 + \cdots + a_{m+1,n}x_n &\leq b_{m+1} \\ x_1, \cdots, x_n &\geq 0 \end{aligned}$$






Bi kasu:

1. **Kasua.** 2. Ereduaren hasierako taulan bideragarritasun primala badago, taula optimoa da

2. **Kasua.** 2. Ereduaren hasierako taulan bideragarritasun primala galdu bada, simplex dual algoritmoa erabili.