

## BASIC SURFACES FOR ENGINEERING

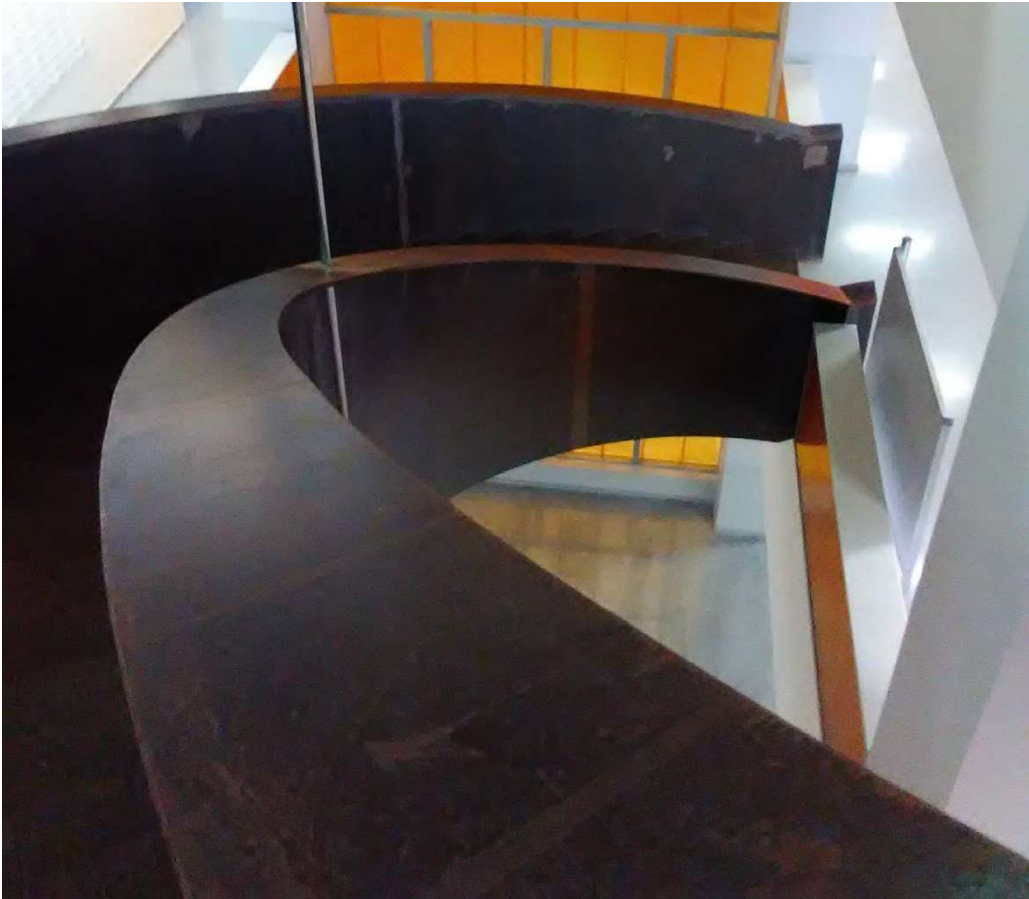


Figure 00. Main stairs of Engineering School of Bilbao II. Picture made by the authors, 2018.

### 3. Flat sections of the surfaces

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### 3. FLAT SECTIONS OF THE SURFACES

At the end of this topic, it is expected that the students will be able to:

- Foresee the geometrical shape that will result from sectioning a surface by a plane.
- Determine the view that allows the simplest or most immediate geometric solution to be obtained.
- Identify the highlights of the section.
- Select the most appropriate plane to perform a section. Application at the intersection of line and surface.
- To develop skills to carry out graphic operations to resolve geometric problems where surfaces are involved.

For an adequate follow-up of this topic it is necessary to have achieved the skills from previous learning topics:

- Surfaces: Representation.
- Dihedral System: Single and double auxiliary projections.
- Definition and representation of the conics.

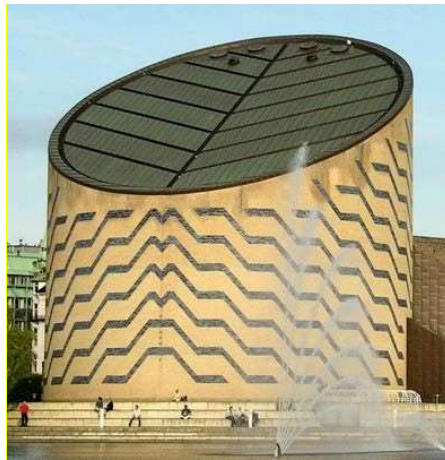


Figure 3.1. Planetarium Copenhagen. <https://www.flickr.com/photos/jimkillock/13929564401>

The study of the subject is focused on the concepts of flat surface sections. The examples shown and the exercises proposed are intended to clarify the concepts without the complications of laborious tracing. Computer aided drawing applications have the ability to do such tedious tracing work.

### 3.1 GEOMETRICAL SHAPE THAT RESULTS FROM SECTIONING A SURFACE BY A PLANE

The section produced by a plane cutting a surface is called a flat section.

In the case of the pyramid and the prism, the flat section is a closed polygon which vertices are on the lateral edges of the surface. It may happen that the secant plane cuts the guideline. Figures 3.2 and 3.3.

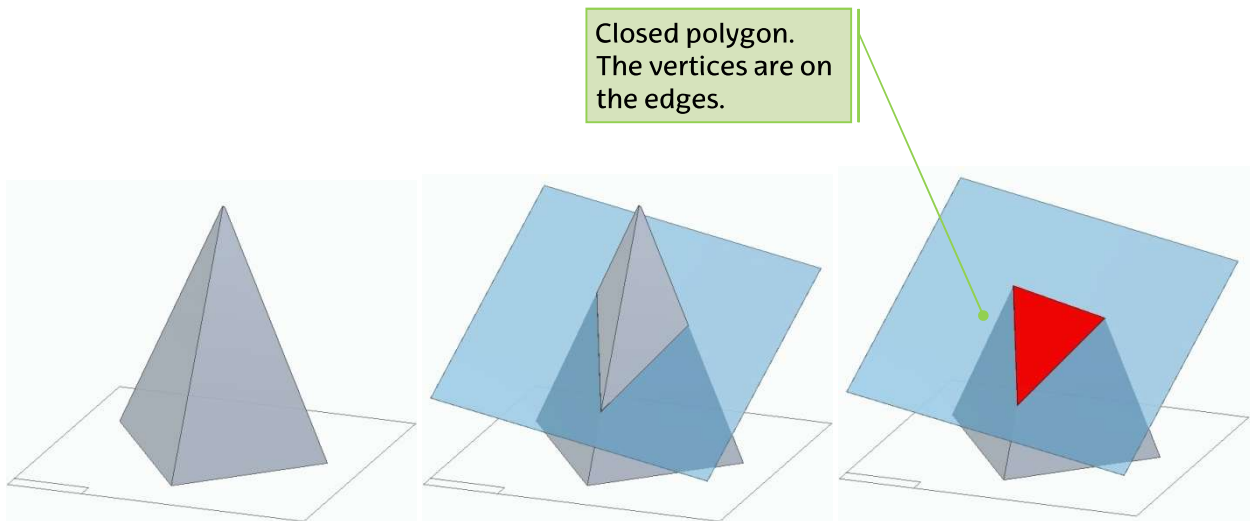


Figure 3.2. Flat section of a pyramid (Image made with Solid Edge)

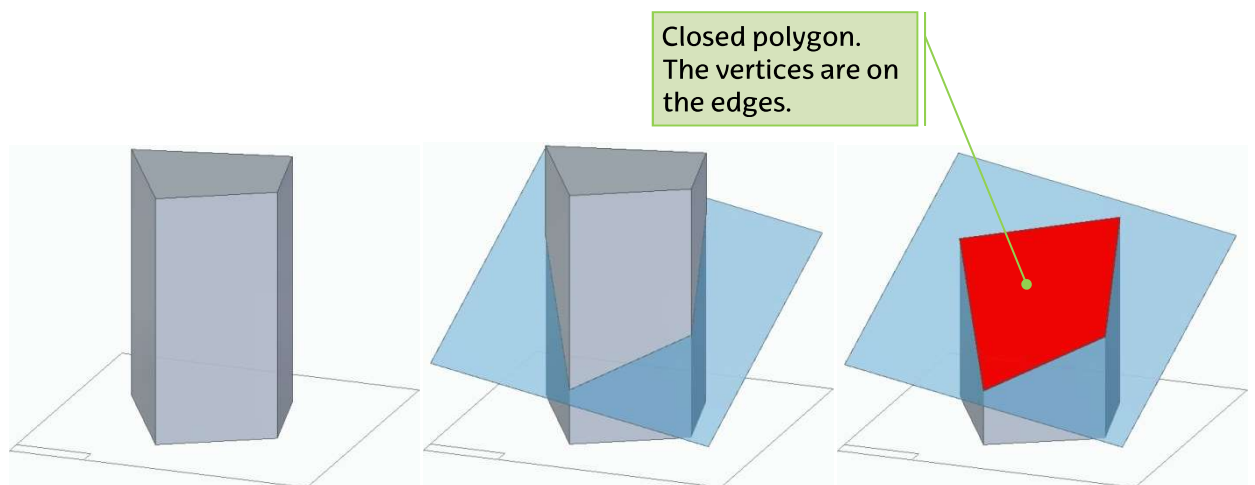


Figure 3.3. Flat section of a prism (Image made with Solid Edge)

In the case of the cone and the cylinder, the flat section is conical. It may happen that the secant plane cuts the guideline, in which case the section line is closed and one of the sides is straight. Figures 3.4 and 3.5.

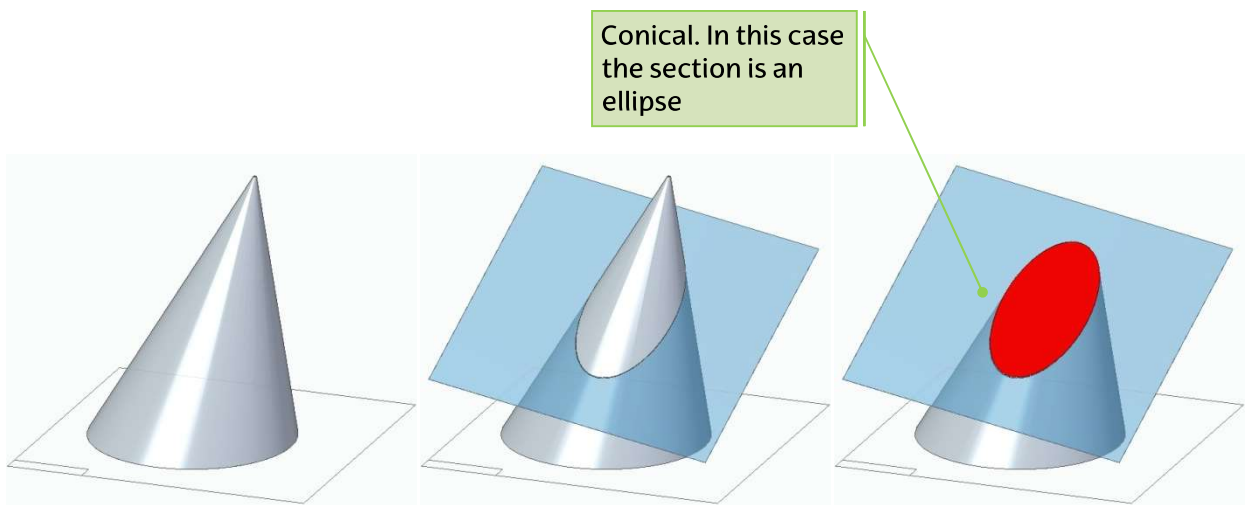


Figure 3.4. Flat section of a cone (Image made with Solid Edge)

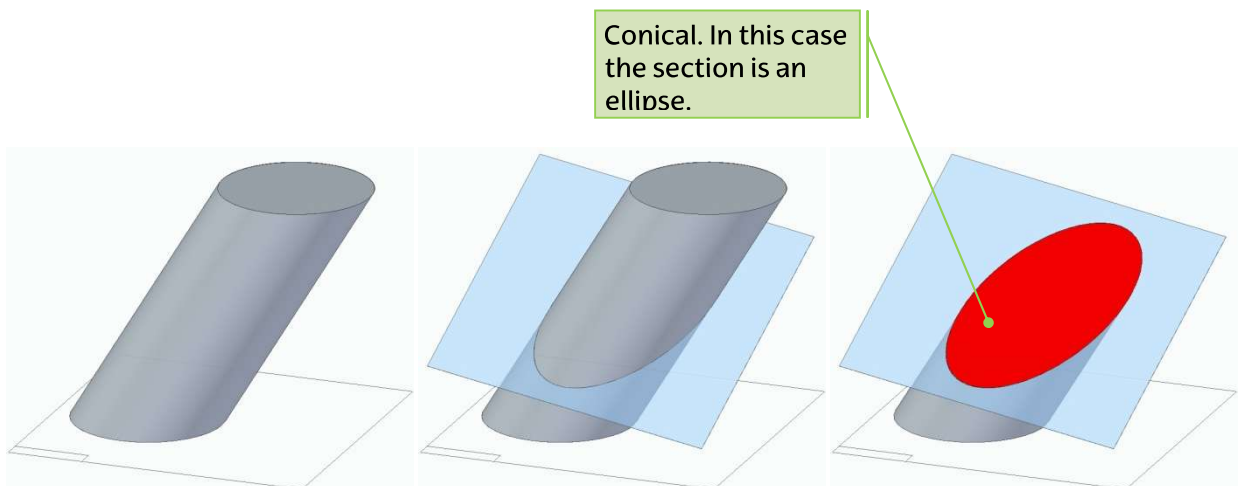


Figure 3.5. Flat section of a cylinder (Image made with Solid Edge)

In the case of the sphere, the flat section is a circle. Figure 3.6.

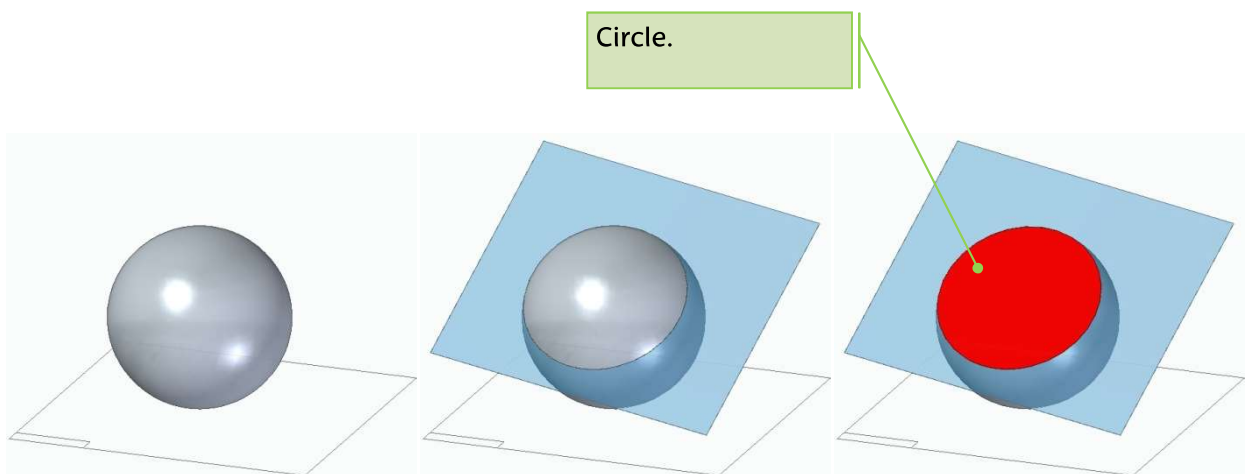


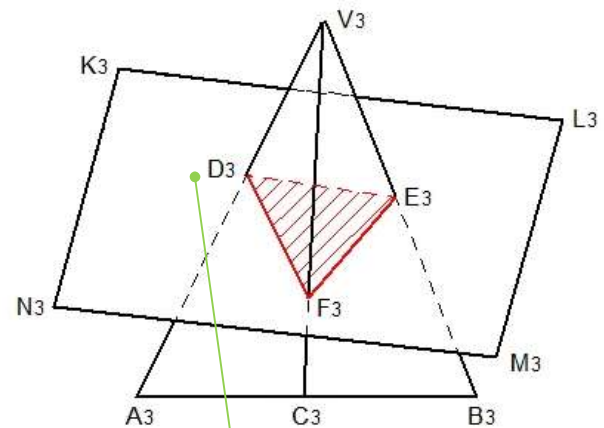
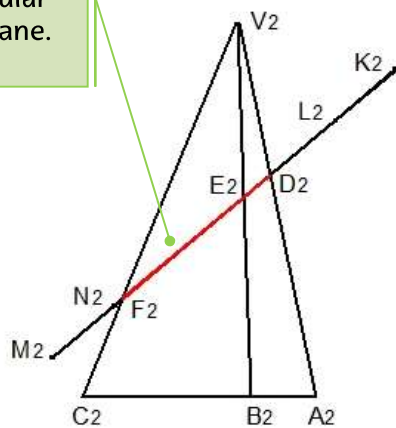
Figure 3.6. Flat section of a sphere (Image made with Solid Edge)

### 3.2 DETERMINE THE VIEW THAT ALLOWS THE SIMPLEST OR MOST IMMEDIATE GEOMETRIC SOLUTION

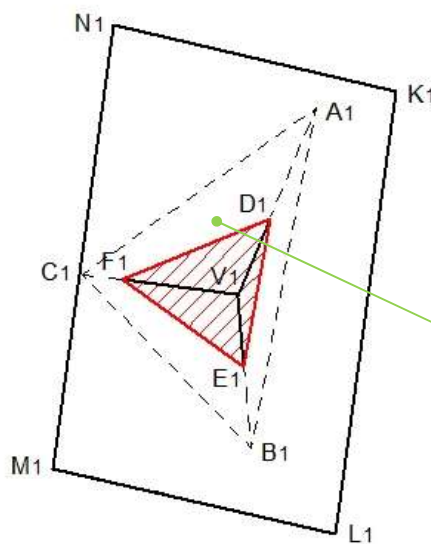
The flat section is found by obtaining the points of intersection between the generating line of the surface and the plane that is cut by.

The favourable position for finding the flat section is when the secant plane is perpendicular to one of the projection planes. In this position, the intersection between the generating lane of the surface and the plane that cuts it is obtained directly. Figure 3.7.

Plane perpendicular to the vertical plane. Immediate



Remarkable points on edges: D, E, F



Remarkable points on edges: D, E, F

Figura 3.7. Section of a surface by a projection plane (Image made with Solid Edge)

When the secant plane is oblique, the favourable situation can be reached by changing the plane. This method is less laborious than the general method, and thus, is the more advisable, even more when the more generatrices are used to find the section. Figure 3.8.

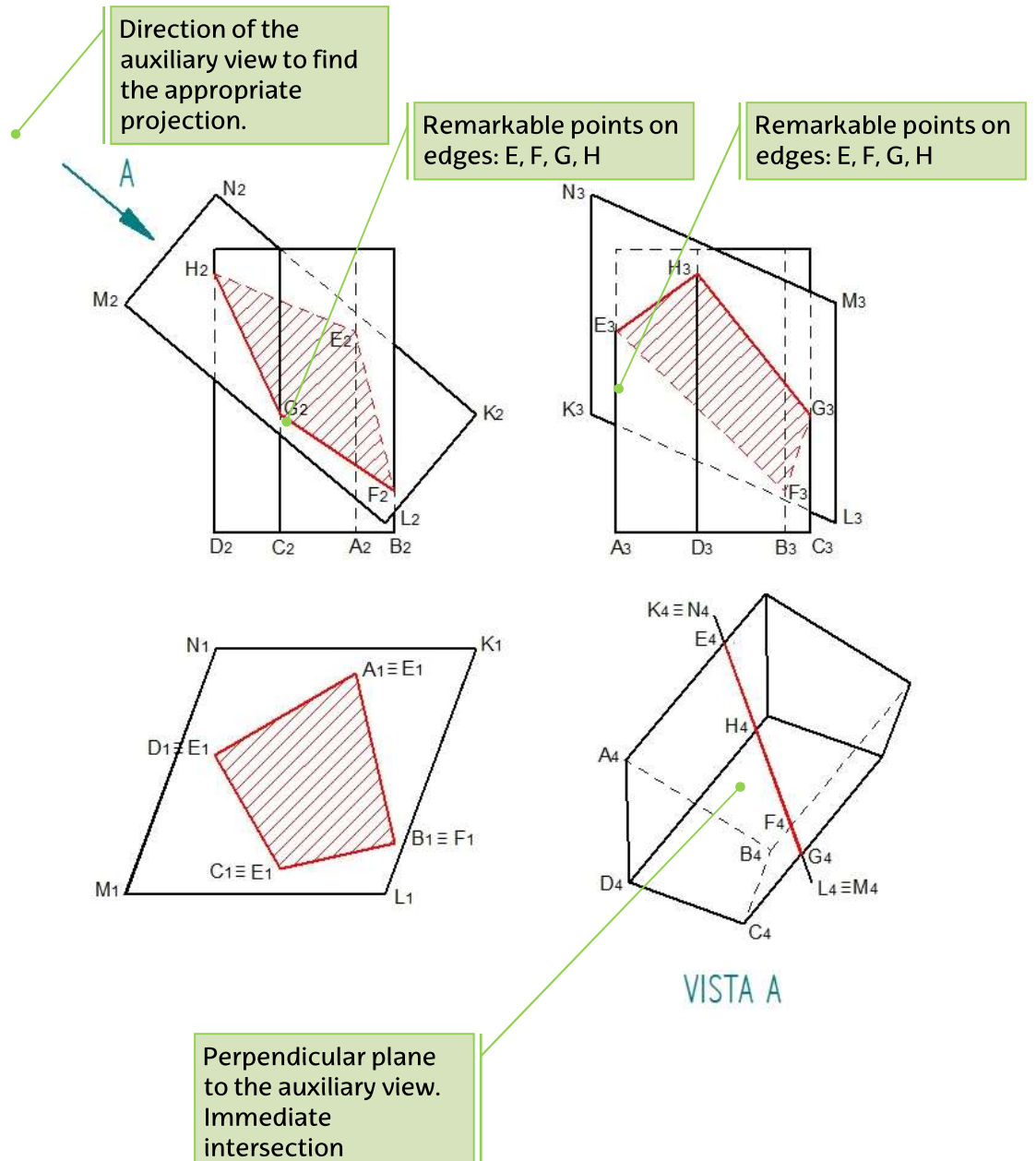


Figure 3.8. Section of a surface by an oblique plane (Image made with Solid Edge)



### 3.3 HIGHLIGHTS OF THE FLAT SECTIONS

The highlights of a flat section are those that provide the key information for the shape of the section.

In the case of the pyramid and the prism, the notable points of the flat section are found on the edges of the surface. Figures 3.7 and 3.8

In the case of the cone and the cylinder, the end points, i.e. those furthest and closest to each of the projection planes, are notable the points of the flat section. Figure 3.9.

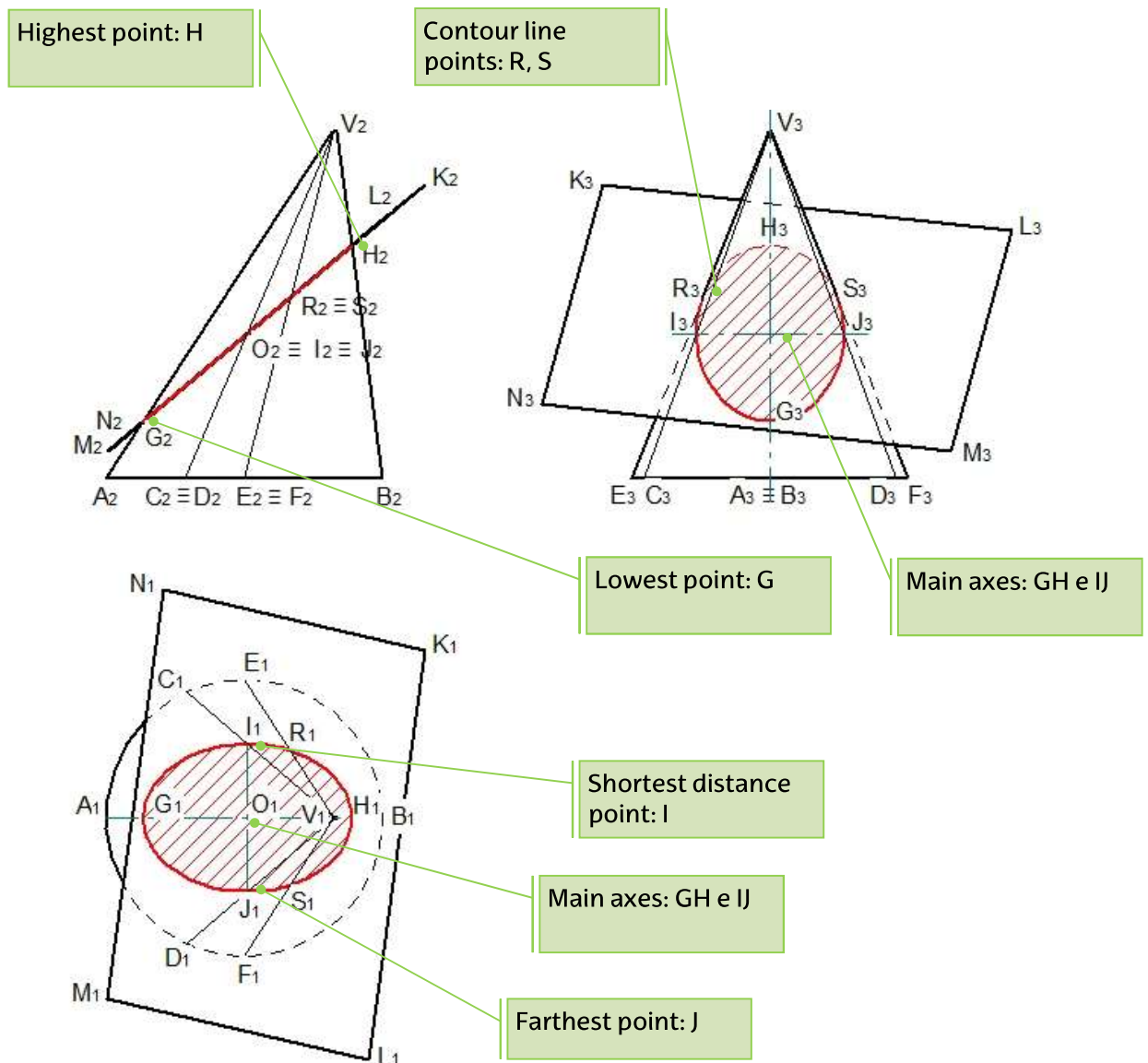


Figure 3.9. Remarkable points of the flat section of a cone (Image made with Solid Edge)

In the particular case where the cone has a plane of symmetry and the secant plane is perpendicular to this plane of symmetry, the end points are the vertices of the main axes of the resulting cone as a section. Figure 3.9.

In the case of the sphere, the resulting circumference as a section is generally projected as an ellipse. The notable points are those that define the main axes of this ellipse. The major axis is on the line parallel to the projection plane that passes through the center of the ellipse, the minor axis is on the line of maximum slope (or maximum inclination) that passes through the center of the ellipse, the axes are perpendicular one to each other in the projection. Figure 3.10.

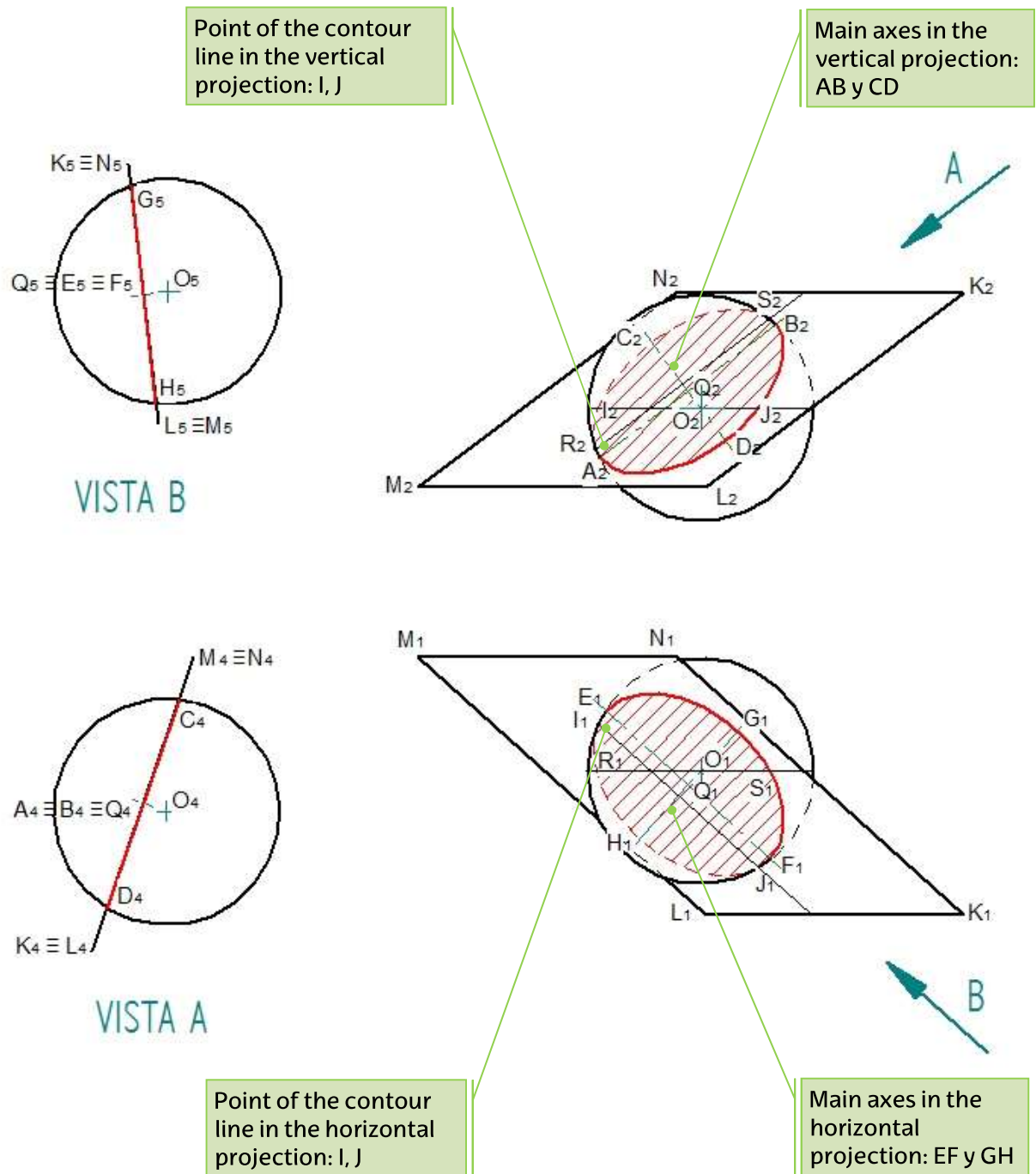


Figure 3.10. Remarkable points of the flat section of a sphere (Image made with Solid Edge)

Noteworthy point are also those found in the contour line guidelines, because they delimit the section that can be visualized in each of the views. In these contour points, the curve of the section is tangential to the guideline of the contour line. Figures 3.9 and 3.10

### 3.4 SPECIFIC FLAT SECTIONS

These are sections that have some specific features that make them different from the general sections.

#### 3.4.1. THE SECANT PLANE PASSES THROUGH THE VERTEX OF THE SURFACE

In the case of the pyramid and the cone, if the secant plane passes through the vertex, the section is formed by two guidelines. Figures 3.11, 3.12 and 3.13.

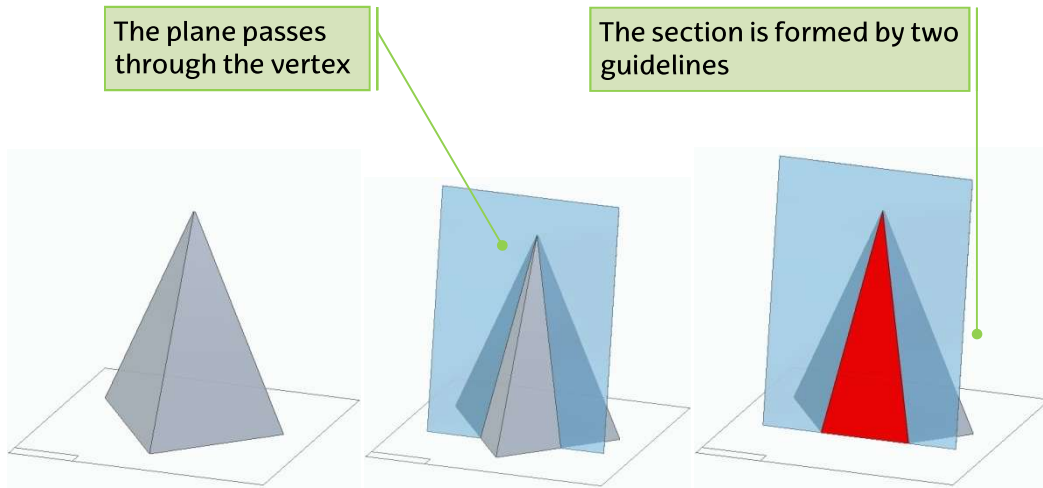


Figure 3.11. Particular flat section of a pyramid (Image made with Solid Edge)

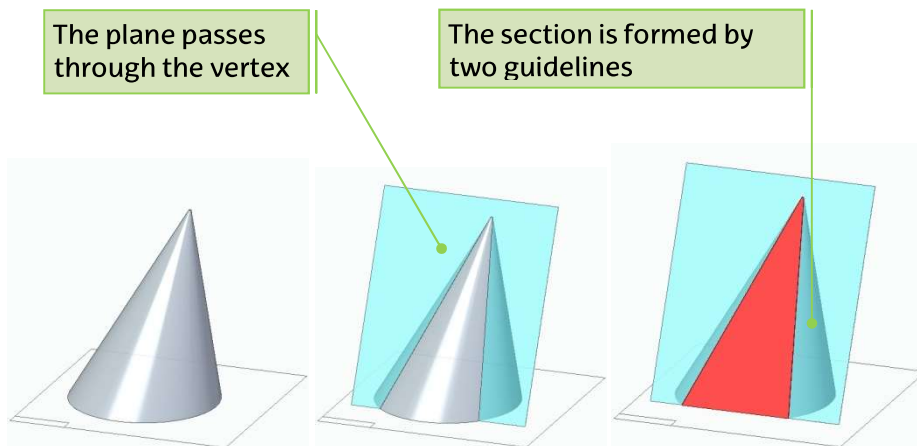


Figure 3.12. Particular flat section of a cone (Image made with Solid Edge)



Figure 3.13. Iberdrola Tower. Mintegui.<http://mintegui.blogspot.com.es/2012/07/torre-iberdrola-debilbao.html>

In the case of the prism and the cylinder, since they do not have a common vertex, it can be considered that the secant plane passes through the vertex when it is parallel to the guidelines, and such section is also formed by two guidelines. Figures 3.14 and 3.15.

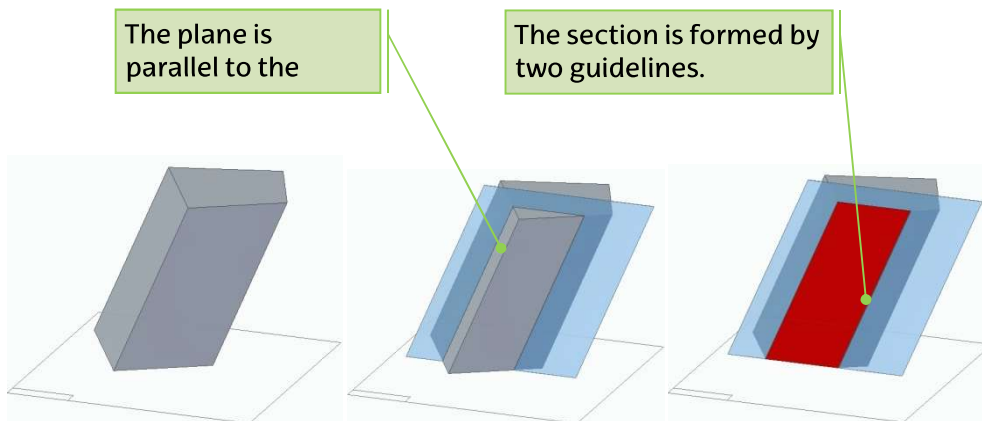


Figure 3.14. Particular flat section of a prism (Image made with Solid Edge)

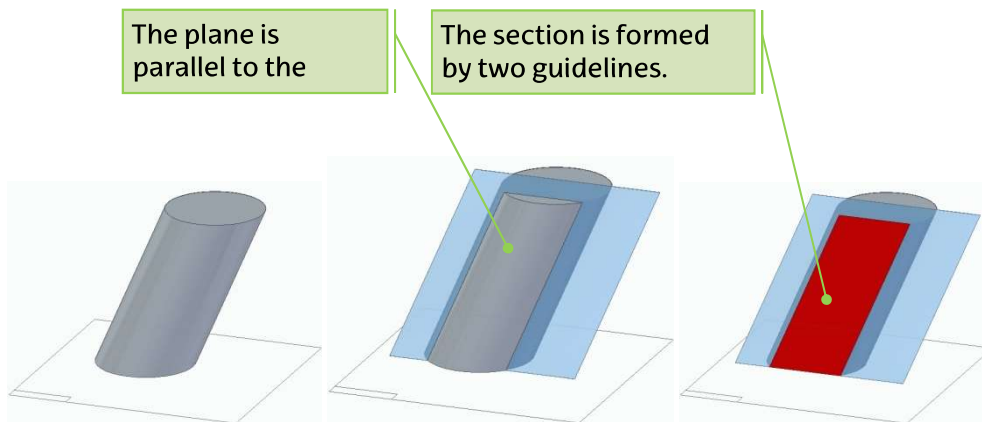


Figure 3.15. Particular flat section of a cylinder (Image made with Solid Edge)

### 3.4.2. THE SECANT PLANE IS PARALLEL TO THE DIRECTRIX

In the case of the prism and the cylinder, the guideline and section are the same. The section layout is obtained by translating the directrix. Figures 3.16 and 3.17

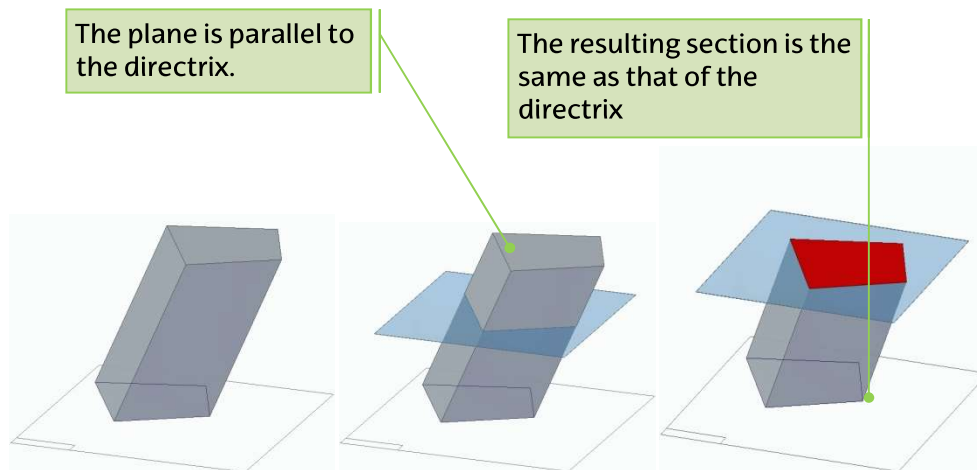


Figura 3.16. Section of a prism by a plane parallel to its base (Image made with Solid Edge)

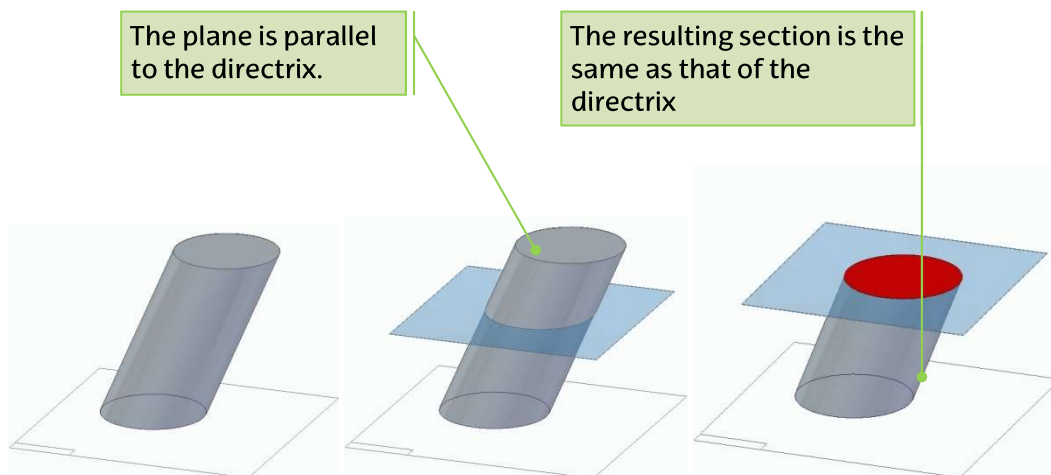


Figure 3.17. Section of a cylinder by a plane parallel to its base (Image made with Solid Edge)

In the case of the pyramid and the cone, the directrix and section are similar. The section layout is obtained by homotecy where, the vertex of the surface is the center of homotecy. Figures 3.18 and 3.19

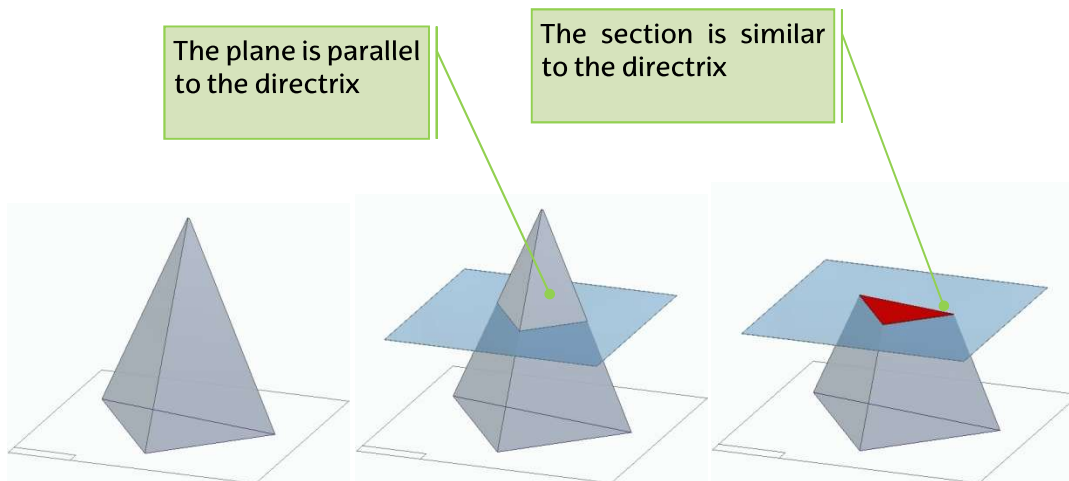


Figure 3.18. Section of a pyramid by a plane parallel to its base (Image made with Solid Edge)

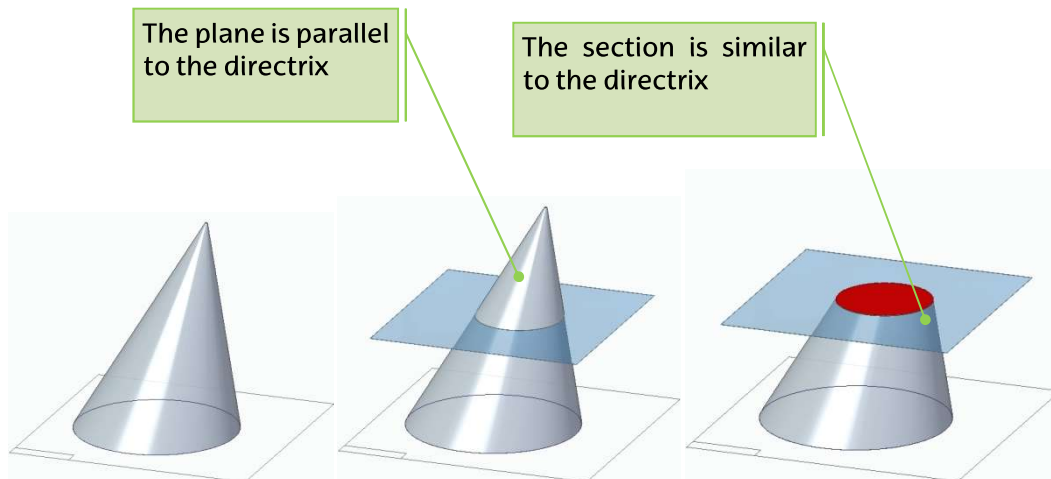


Figure 3.19. Section of a cone by a plane parallel to its base (Image made with Solid Edge)





This also happens when the surface is cut by two parallel planes, even if they are not parallel to the directrix. Figure 3.20.

In the case of the prism and the cylinder, the two sections are equal one to each other. In the case of the pyramid and the cone, the two sections are similar one to each other.

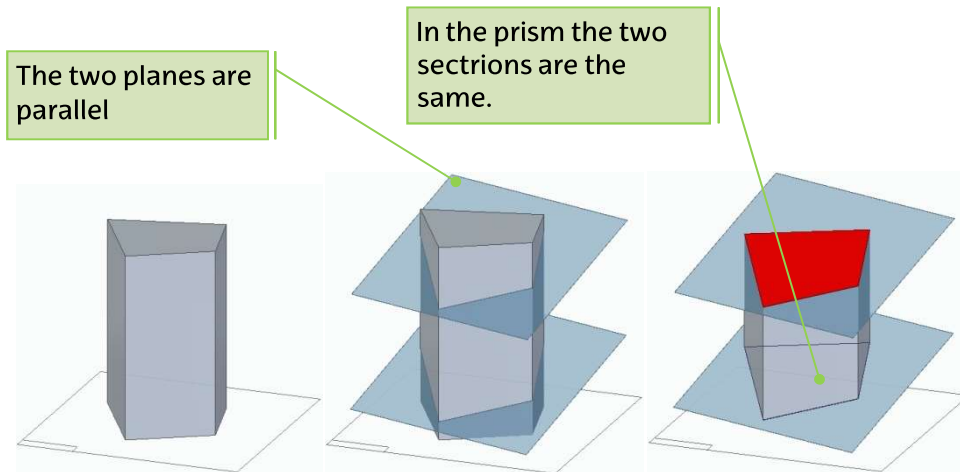


Figure 3.20. Section of a prism by two planes parallel to each other (Image made with Solid Edge)

### 3.4.3. STRAIGHT SECTION

The section produced by a plane perpendicular to the guideline of the prism and the cylinder is called a straight section. Figure 3.21.

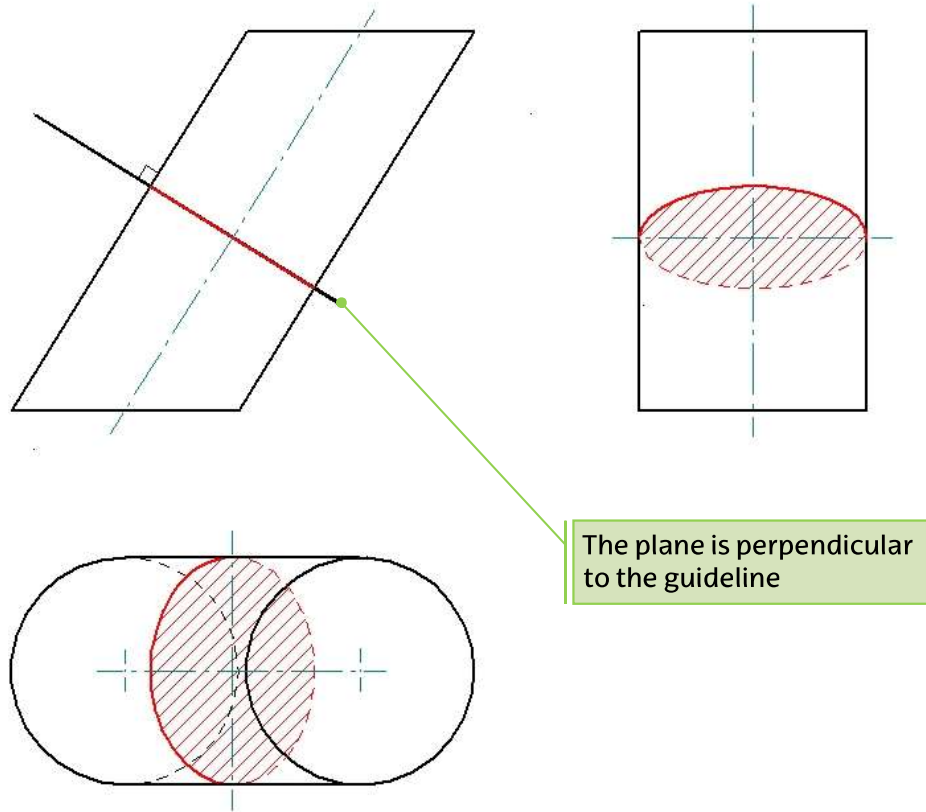


Figure 3.21. Straight section of a cylinder (Image made with Solid Edge)

All straight sections of the same prismatic and cylindrical surface are equal, since they are all parallel one to each other. See figure 3.20.

### 3.4.1. THE CONIC CURVES

On radiated surfaces the guideline of a circle or an ellipse, the secant plane not passing through the vertex, produces a conical section. The type of cone depends on the position of the secant plane in relation to the surface.

This study is carried out only for conical and cylindrical surfaces, both of revolution: straight surfaces of circular guideline.

#### Elliptical section

This type of section is produced when the secant plane cuts all the guidelines of surface. It can be produced on both conical and cylindrical surfaces. Figures 3.22, 3.23 and 3.24.

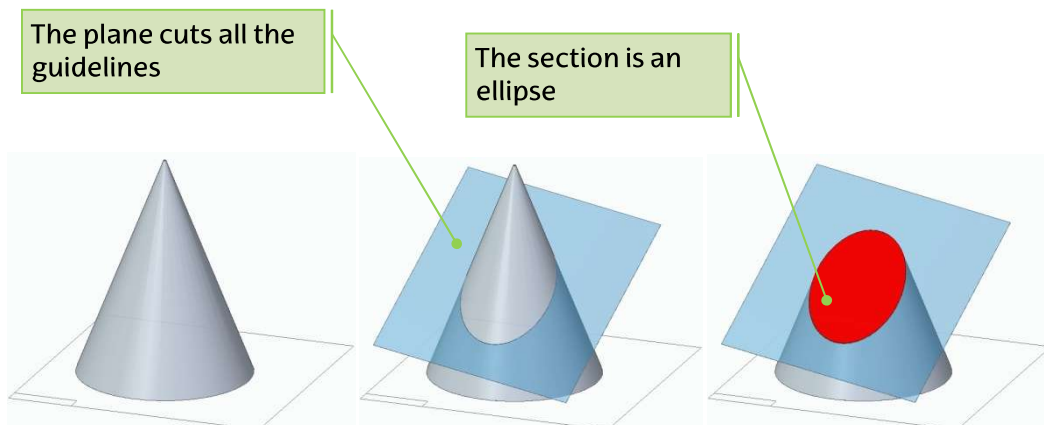


Figure 3.22. Section of all cone generators by a plane (Image made with Solid Edge)

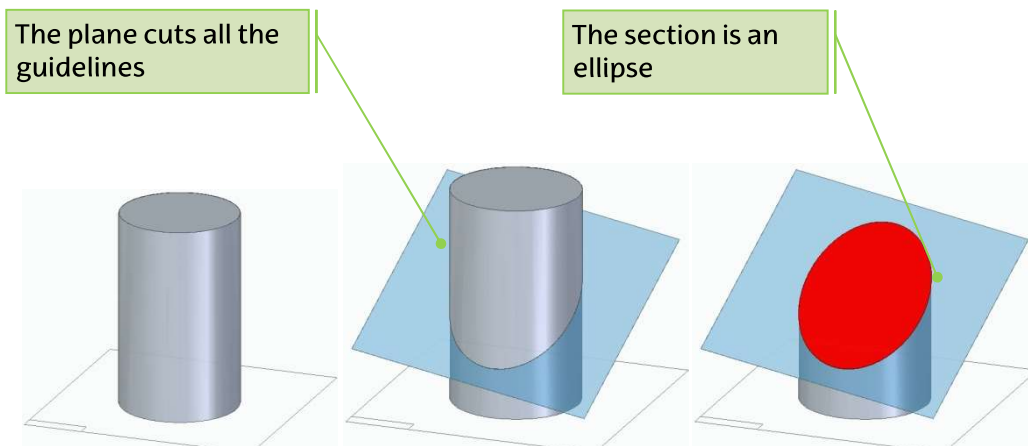


Figure 3.23. Section of all the generatrices of a cylinder by a plane (Image made with Solid Edge)

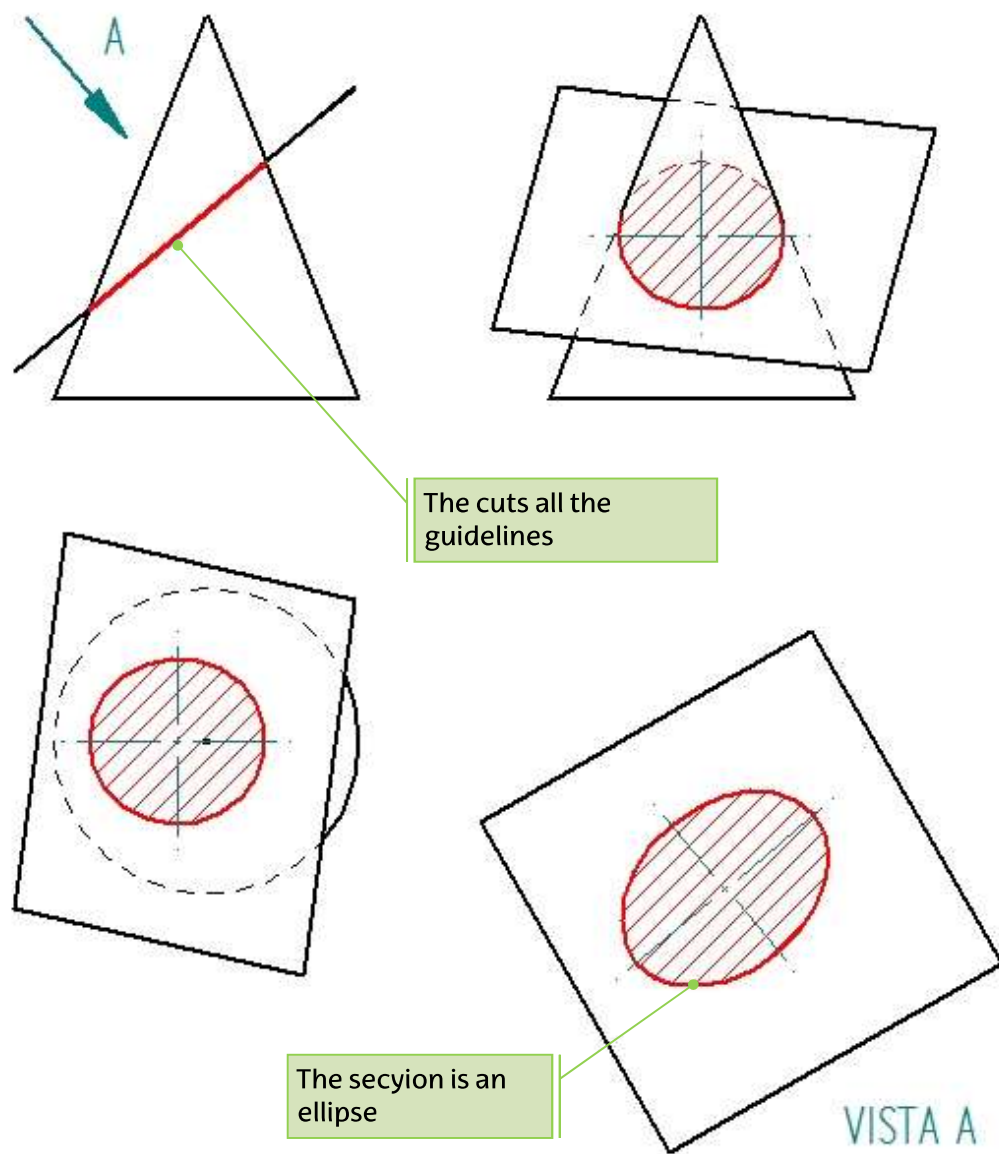


Figura 3.24. Section of all cone generators by a plane (Image made with Solid Edge)

When the secant plane is parallel to the guideline, the section is a circle. See figures 3.17 and 3.19.

## Parabolic section

This type of section is the result in conical surfaces when the secant plane is parallel to a surface guideline. Figures 3.25 and 3.26

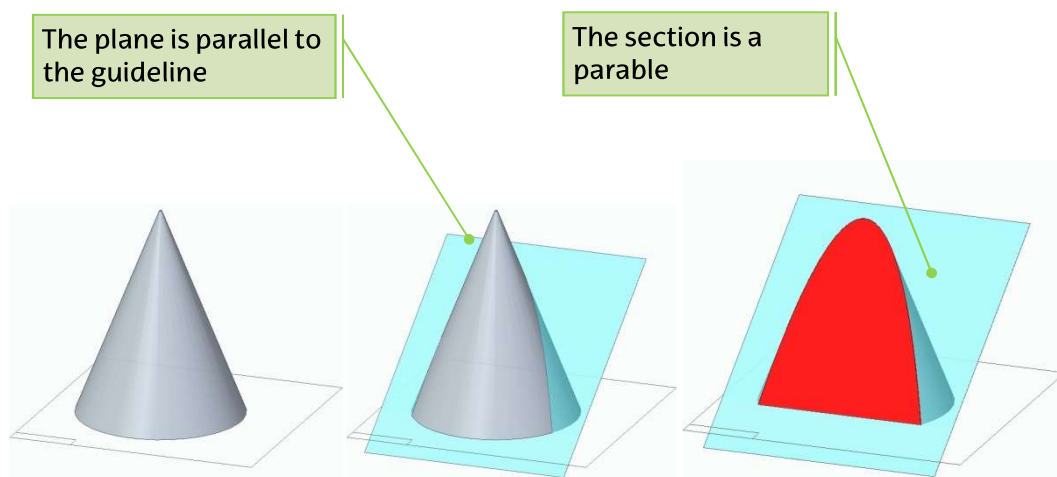


Figure 3.25. Section of a cone cut by a plane parallel to a generator (Image made with Solid Edge)

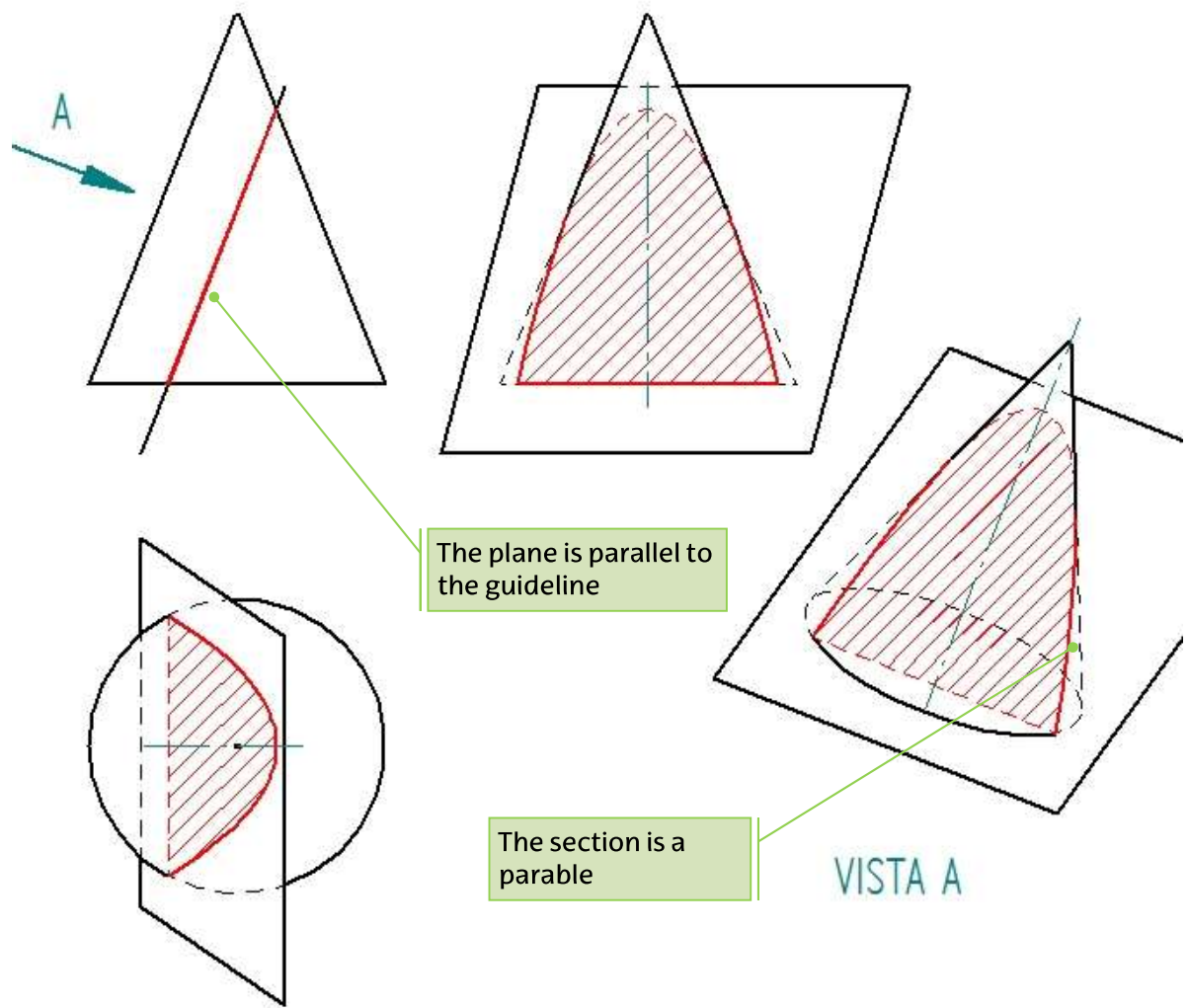


Figure 3.26. Section of a cone cut by a plane parallel to a generator (Image made with Solid Edge)

## Hyperbolic section

This type of section occurs on conical surfaces when the secant plane is parallel to two surface guideline. Figures 3.27 and 3.28

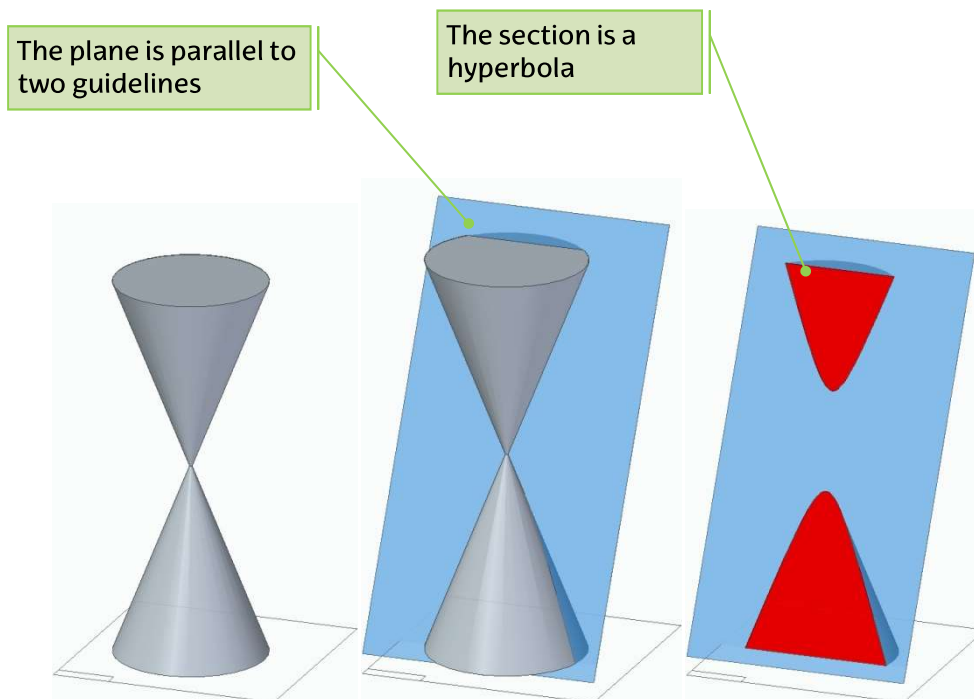


Figure 3.27. Section of a cone by a plane parallel to two generatrices (Image made with Solid Edge)

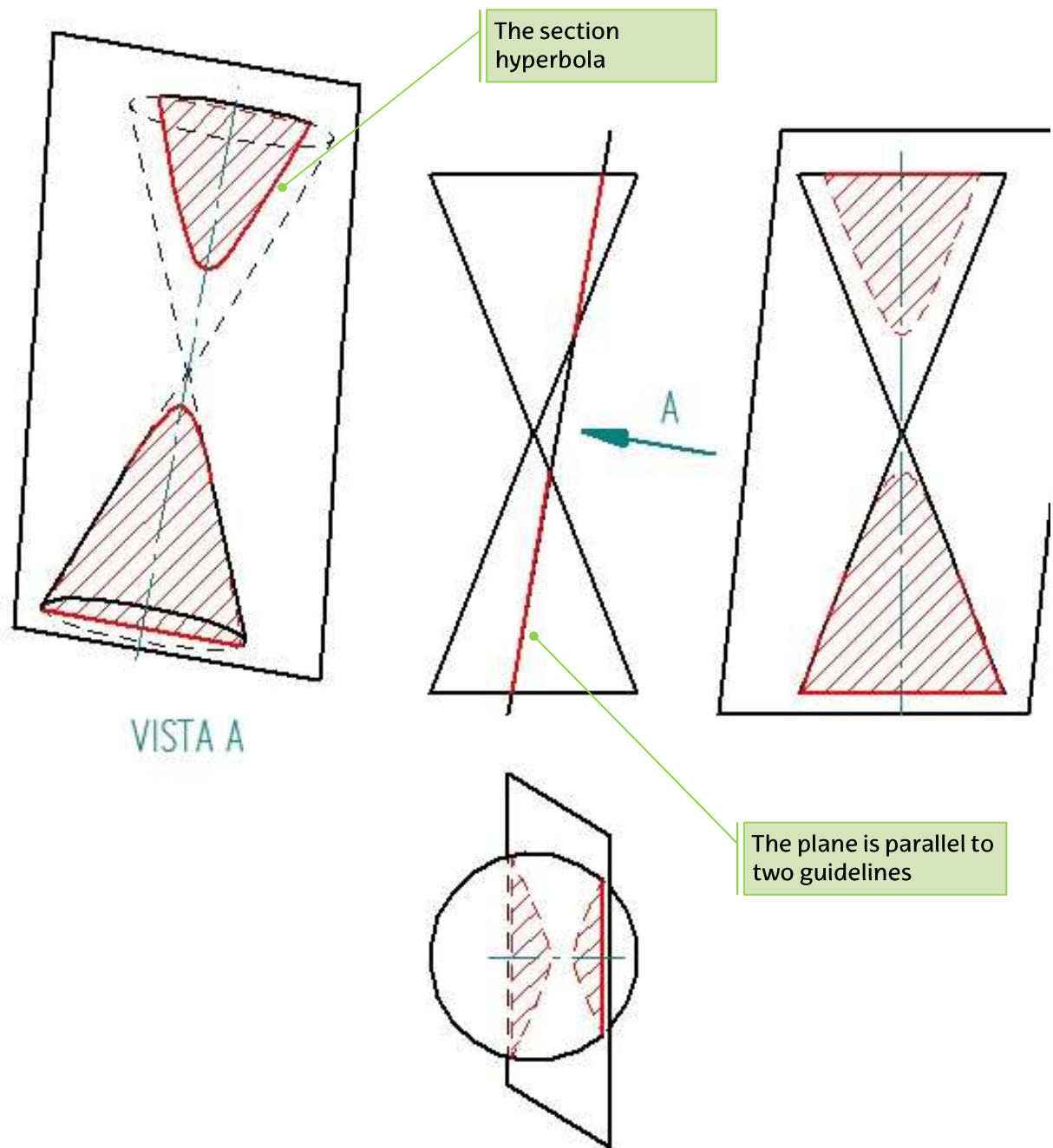


Figure 3.28. Section of a cone by a plane parallel to two generatrices (Image made with Solid Edge)



### 3.5 POINTS OF INTERSECTION OF A LINE WITH A SURFACE

A line intersects a surface with one or more points.

The maximum number of points of intersection of a line with a surface indicates the order of the surface. For example, a plane is a first-order surface. Convex radiated surfaces and the spherical surfaces are second order planes which are also called quadratics.

The general procedure for determining the points of intersection is to find the flat section that produces a plane containing the given line. The common points of the line and the section are the sought-after points. Figure 3.29.

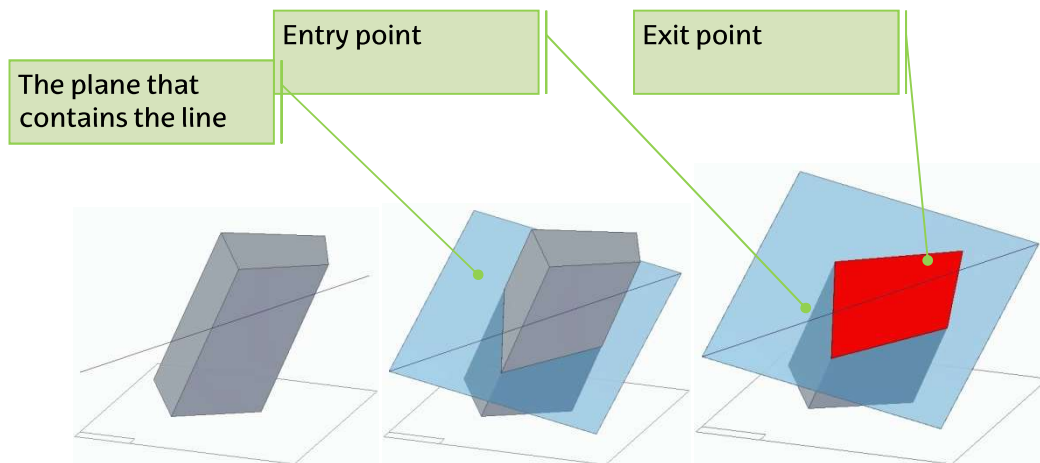


Figure 3.29. Intersection of a line with a prism (Image made with Solid Edge)

Since an infinite number of planes pass along a straight line, the selection of the plane will be carried out pursuing a favourable situation or that produces an easily traced section. In the case of the prism, the pyramid and the sphere would be those planes perpendicular to the planes of projection, see figure 3.7. In the case of the cylinder and cone would be those planes that go through the vertex of the surface, see Figures 3.12 and 3.15.