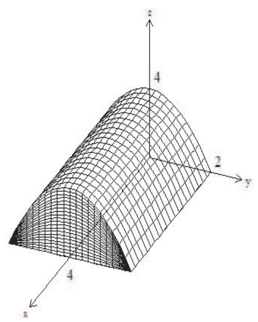
Gaiak 6 - 11: 7. Autoebaluaketa

1 Ariketak

1 Ariketa. a eta b alboko aldeak dituen paralelogramo baten azalera, hurrengo funtzioaren bidez emanda dator:
$A = ab\sin\theta$, non θ alde horiek osatzen duten angelua den.
a) Kalkulatu A-ren erritmo aldaketak a-rekiko eta b-rekiko, $a = 10, b = 20$ eta $\theta = \pi/2$ izanik.
b) Demagun a, b eta θ denboran aldatzen direla: $a = 2t, b = 4t, \theta = t\pi/10$. Kalkulatu A-ren erritmo aldaketa t-rekiko aurreko a, b eta θ balioetarako.

2 Ariketa. Izan bedi irudiko zilindro parabolikoa. Kalkulatu bere grabitate zentroa, puntu bakoitzeko dentsitatea, $d(x, y, z)$, konstantea dela suposatuz.



3 Ariketa. Ebatzi hurrengo EDA, $F(x, y) = x^n y^m$ moduko faktore integratzailea kalkulatuz:

$$(3xy^2 - 5x^3y)dx + (4x^2y - 3x^4)dy = 0$$

4 Ariketa. Izan bedi $z = x^2 + y^2, z = 4$ gainazalek definitutako S solidoa. S-ren adierazpen grafikoa egin eta kalkulatu bere bolumena.

5 Ariketa. Laplace-en transformatua erabiliz, ebatzi ondorengo problema:

$$y'' + 2y' + 5y = \mathrm{e}^{-t}\sin t$$

$$y(0) = 0$$

$$y'(0) = 1$$

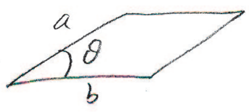
6 Ariketa. Koefiziente mugagabeen metodoa erabiliz, ebatzi hurrengo EDA:

$$y'' + y' - 2y = 2x^2 - 3x$$

2 Ebazpenak

1 Ebazpena.

$$A(a,b,\theta) = ab\sin\theta$$



a) $A_a = b\sin\theta,\ A_a(10,20,\frac{\pi}{2}) = 20$

$$A_b = a\sin\theta,\ A_b(10,20,\pi/2) = 10$$

b) $A(a(t),b(t),\theta(t))$

$$A'(t) = A_a a'(t) + A_b b'(t) + A_\theta \theta'(t) = 2b\sin\theta + 4a\sin\theta + \frac{\pi}{10}ab\cos\theta$$

non $t = 5, a = 10, b = 20, \theta = \frac{\pi}{2}$

$$A'(5) = 40 + 40 + \frac{\pi}{10}\cdot 0 = 80$$

2 Ebazpena.

$$z(x,y) = 4 - y^2,\ y \in [-2,2], x \in [0,4]$$

$d = k$ balio konstanterako:

$$M = k\int\int\int_V dxdydz = k\int_0^4\int_{-2}^2\int_0^{4-y^2} dzdydx = k\int_0^4\int_{-2}^2 (4-y^2)dydx =$$

$$= k\int_0^4 \frac{32}{3}dx \approx 42.67K$$

$$k\int\int\int_V xdxdydz \approx 85.33K;\ \int\int\int_V ydxdydz = 0$$

$$k\int\int\int_V zdxdydz = 68.267K$$

$$x_c = \left(\frac{85.33}{42.67}, 0, \frac{68.267}{42.67}\right) \approx (2, 0, 1.6)$$

3 Ebazpena.

$$(3xy^2 - 5x^3y)dx + (4x^2y - 3x^4)dy = 0$$

$F = x^n y^m$ funtzioaz biderkatzen badugu:

$$(3x^{n+1}y^{m+2} - 5x^{n+3}y^{m+1})dx + (4x^{n+2}y^{m+1} - 3x^{n+4}y^m)dy = 0$$

$$M_y = N_x$$

$$3(m+2)x^{n+1}y^{m+1} - 5(m+1)x^{n+3}y^m = 4(n+2)x^{n+1}y^{m+1} - 3(n+4)x^{n+3}y^m$$

$$3(m+2)y - 5(m+1)x^2 = 4(n+2)y - 3(n+4)x^2$$

$$y(3m - 4n - 2) + x^2(3n - 5m + 7) = 0$$

$$3m - 4n = 2 \quad \wedge \quad -5m + 3n = -7 \;\Rightarrow\; m = 2 \quad \wedge \quad n = 1 \;\Rightarrow\; F = xy^2$$

$$(3x^2y^4 - 5x^4y^3)dx + (4x^3y^3 - 3x^5y^2)dy = 0 \quad \text{zehatza}$$

$$\exists u(x,y)/u_x = 3x^2y^4 - 5x^4y^3 \quad (1) \quad \wedge \quad u_y = 4x^3y^3 - 3x^5y^2 \quad (2)$$

$$(1) \;\Rightarrow\; u(x,y) = x^3y^4 - x^5y^3 + h(y) \quad (3)$$

$$(3) \;\Rightarrow\; u_y = 4x^3y^3 - 3x^5y^2 + h'(y)$$

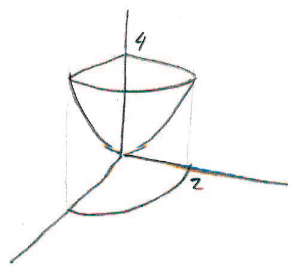
$$(2) + (3) \;\Rightarrow\; h'(y) = 0 \;\Rightarrow\; h(y) = C$$

Beraz, soluzio orokorra:

$$x^3y^4 - x^5y^3 = C$$

4 Ebazpena.

$$V = 4\int_0^{\pi/2}\int_0^2 \rho^3 d\rho d\theta = 16\int_0^{\pi/2} d\theta = 8\pi$$



5 Ebazpena.

$$y'' + 2y' + 5y = e^{-t}\sin t, \quad \alpha(y(t)) = Y(s)$$

$$y(0) = 0, \quad y'(0) = 1$$

$$s^2Y - 1 + 2sY + 5Y = \alpha(e^{-t}\sin t) = \frac{1}{(s+1)^2+1}$$

$$(s^2 + 2s + 5)y = 1 + \frac{1}{(s+1)^2+1}$$

$$Y(s) = \frac{1}{s^2+2s+5} + \frac{1}{(s^2+2s+5)((s+1)^2+1)}$$

$$s^2 + 2s + 5 = (s+1)^2 + 4$$

$$\frac{1}{(s^2+2s+5)(s^2+2s+2)} = \frac{As+B}{s^2+2s+5} + \frac{Cs+D}{s^2+2s+2}$$

$$\Rightarrow \quad A = C = 0, \; B = -1/3, D = 1/3$$

$$y(t) = \mathscr{L}^{-1}(Y(s)) = \frac{-1}{3}\mathscr{L}^{-1}\left(\frac{1}{(s+1)^2+4}\right) + \frac{1}{3}\mathscr{L}^{-1}\left(\frac{1}{(s+1)^2+1}\right) +$$

$$+\mathscr{L}^{-1}\left(\frac{1}{(s+1)^2+4}\right) = \frac{1}{3}e^{-t}\sin t + \frac{1}{3}e^{-t}\sin(2t)$$

6 Ebazpena.

$$y'' + y' - 2y = 2x^2 - 3x \quad \Rightarrow \quad m^2 + m - 2 = 0 \quad \Rightarrow \quad m = 1, -2$$

$$y_{GH} = C_1 e^{x} + C_2 e^{-2x}$$

$$y_{PC} = y = Ax^2 + Bx + C \quad \Rightarrow \quad y' = 2Ax + B \quad \Rightarrow \quad y'' = 2A$$

$$2A + 2Ax + B - 2Ax^2 - 2Bx - 2C = 2x^2 - 3x$$

$$-2Ax^2 + x(2A - 2B) + 2A + B - 2C = 2x^2 - 3x$$

$$A = -1, B = 1/2, C = -3/4$$

$$y_{GC} = C_1\mathrm{e}^x + C_2\mathrm{e}^{-2x} - x^2 + \frac{1}{2} - \frac{3}{4}$$