## 10. Kapitulua

# Laplace-ren transformatua

**10.1. Ariketa** Lortu funtzio hauen Laplaceren transformatua.

1. $f(t) = t^2 e^t$
2. $f(t) = e^{2t}(\cos 6t - 5\sin 6t)$
3. $f(t) = \sin^2 t$
4. $f(t) = \begin{cases} 0 & t < a \quad \text{denean} \\ 1 & t > a \quad \text{denean} \end{cases}$

**10.2. Ariketa** Funtzio hauen Laplaceren alderantzizko transformatua lortu.

1. $F(s) = \frac{1}{(s-1)(s-2)}$
2. $F(s) = \frac{e^{-5s}}{(s-2)^4}$
3. $F(s) = \frac{2s^2-4}{(s-1)(s-2)(s-3)}$
4. $F(s) = \frac{6s-4}{s^2-4s+20}$
5. $F(s) = \frac{4s+12}{s^2+8s+16}$
6. $F(s) = \frac{1}{(s-2)^3}$
7. $F(s) = \frac{3s+1}{(s-1)(s^2+1)}$
8. $F(s) = \frac{3s+7}{s^2-2s-3}$

**10.3. Ariketa** Laplaceren transformatua erabiliz, ebatzi problema hauek.

1. $y'' + 9y = \cos 2t, \qquad y(0) = 1, y(\pi/2) = -1$
2. $y'' - 3y' + 2y = 4e^{2t}, \qquad y(0) = 3, y'(0) = 5$
3. $y'' + 2y' + 5y = e^{-t}\sin t, \qquad y(0) = 0, y'(0) = 1$
4. $y''' - 3y'' + 3y' - y = t^2 e^t, \qquad y(0) = 1, y'(0) = 0, y''(0) = -2$

**10.4. Ariketa** Laplaceren transformatua erabiliz, EDA honen soluzio orokorra lortu.

$$y''' - 3y'' + 3y' - y = t^2 e^t$$

**10.5. Ariketa** Laplaceren transformatua erabiliz, problema honen soluzioa aurkitu.

$$ty'' - ty' - y = 0$$
$$y(0) = 0$$
$$y'(0) = 3$$

**10.6. Ariketa** Laplaceren transformatua erabiliz, problema hauek ebatzi.

1. $\begin{cases} x' = 2x - 3y \\ y' = y - 2x \end{cases} \qquad x(0) = 8, y(0) = 3$

2. $\begin{cases} y' + 2z' = t \\ y'' - z = \mathrm{e}^{-t} \end{cases} \qquad y(0) = 3, y'(0) = -2, z(0) = 0$

3. $\begin{cases} x'' + y' + 3x = 15\mathrm{e}^{-t} \\ y'' - 4x' + 3y = 15 \sin 2t \end{cases} \qquad x(0) = 35, x'(0) = -48, y(0) = 27, y'(0) = -55$

4. $\begin{cases} x' + y' = y + z \\ y' + z' = x + z \\ x' + z' = x + y \end{cases} \qquad x(0) = 2, y(0) = -3, z(0) = 1$