

4. gaia

Ariketak.

1. Egiaztatu emandako funtzioak ondoko ekuazioen soluzioak diren ala ez:

a) $y'' = x^2 + y^2$, $y = 1/x$.

(Em.: Ez)

b) $(x + y)dx + xdy = 0$, $y = (K^2 - x^2)/2x$.

(Em.: Bai)

c) $x''(t) + \omega^2 x(t) = 0$, $x(t) = K_1 \cos(\omega t) + K_2 \sin(\omega t)$.

(Em.: Bai)

d) $y'' - (\lambda_1 + \lambda_2)y' + \lambda_1\lambda_2 y = 0$, $y = K_1 e^{\lambda_1 x} + K_2 e^{\lambda_2 x}$.

(Em.: Bai)

2. Aurkitu ondoko ekuazioen soluzio orokorrak:

a) $xy' - y = y^3$.

(Em.: $y(x) = Kx\sqrt{1 + y^2(x)}$)

b) $3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$.

(Em.: $y(x) = \arctan(K|e^x - 1|^3)$)

c) $y' \tan x = y$.

(Em.: $y(x) = K \sin x$)

d) $(1 + e^x)y'y = e^x$, baldin $y(0) = 1$ bada.

(Em.: $y(x) = \sqrt{1 + 2 \ln[(1 + e^x)/2]}$)

e) $y' = -\frac{x + y}{x}$.

(Em.: $y(x) = \frac{K - x^2}{2x}$)

f) $y dx + (\sqrt{4xy} - x) = 0$.

$$(Em.: \ln|y| + \sqrt{x/y} = K \quad)$$

3. Aurkitu lehenengo mailako ekuazio hauen soluzio orokorrak:

a) $(3x^2 + 6xy^2)dx + (6x^2y + 4y^3)dy = 0.$

$$(Em.: x^3 + 3x^2y^2(x) + y^4(x) = K \quad)$$

b) $(x + y)dx + (x + 2y)dy = 0.$

$$(Em.: 2y^2 + 2xy(x) + x^2 = K \quad)$$

c) $x dx + y dy = \frac{x dy - y dx}{x^2 + y^2}.$

$$(Em.: x^2 + 2 \arctan \frac{x}{y(x)} + y^2(x) = K \quad)$$

d) $(x^2 + 2xy^3)dx + (y^2 + 3x^2y^2)dy = 0.$

$$(Em.: y^3(x) = \frac{K - x^3}{3x^2 + 1} \quad)$$

e) $x \ln(x/y)dy - ydx = 0.$

$$(Em.: \frac{|y(x)|}{|1 + \ln|y(x) - \ln|x||} = K \quad)$$

f) $e^{x-y}y' = 1$, baldin $y(1) = 1$ bada.

$$(Em.: y(x) = x \quad)$$

g) $2xy' = y$, baldin $y(1) = 1$ bada.

$$(Em.: |y(x)| = \sqrt{|x|} \quad)$$

4. Ondoko ekuazio diferentzial zehatzak ala zehatzetarako labur-garriak dira. Askatu itzazu zuzenean ala soilik aldagai baten menpean dagoen faktore integratzailea erabiliz:

a) $(x + y^2)dx - 2xydy = 0.$

$$(Em.: -\frac{y^2}{x} + \frac{1}{x} = K, \quad (\mu(x) = x^{-2}))$$

b) $x dx + y dy = \frac{x dy - y dx}{x^2 + y^2}$

c) $(2xy^2 - 3y^3)dx + (7 - 3xy^2)dy = 0.$

$$(Em.: x^2 - 3xy - \frac{7}{y} = K, \quad (\mu(y) = y^{-2}))$$

d) $(x + \sin x + \sin y)dx + \cos y dy = 0.$

$$(Em.: e^x (\sin y + (x - 1) + (\sin x - \cos x)/2) = K, \quad (\mu(x) = e^x))$$

5. Aurkitu lehenengo mailako ekuazio hauen soluzio orokorrak:

a) $xy' - \frac{y}{x+1} - x = 0.$

$$(Em.: y(x) = \left| \frac{x}{x+1} \right| (K + x + \ln|x|) \quad)$$

b) $y' + y \cos x = \sin x \cos x.$

$$(Em.: y(x) = K e^{-\sin x} + \sin x - 1 \quad)$$

c) $(1 - x^2)y' + xy = a.$

$$(Em.: y(x) = K \sqrt{|1 - x^2|} + ax \quad)$$

d) $y' - y \frac{2x-1}{x^2} = 1.$

$$(Em.: y(x) = K x^2 e^{1/x} + x^2 \quad)$$

6. Askatu ordena desberdineko ekuazio hauek:

a) $xy'' + y' = 0.$

$$(Em.: y(x) = K_1 \ln|x| + K_2 \quad)$$

b) $xy''' + y'' = 1 + x.$

$$(Em.: y(x) = K_1 x (\ln|x| - 1) + \frac{x^2}{2} + \frac{x^3}{12} + K_2 x + K_3 \quad)$$

c) $yy'' - y'(1 + y') = 0.$

$$(Em.: y(x) = \frac{1}{K_1} + K e^{K_1 x} \quad)$$

d) $yy' = y'' \sqrt{y^2 + (y')^2} - y'y'' = 0.$

$$(Em.: y(x) = \frac{1 - K_2 e^x}{K_1 (1 + K_2 e^x)} \quad)$$

e) $xy'' = y'$, baldin $y(0) = 0$ eta $y'(0) = 0$ badira.

$$(Em.: y(x) = K_1 x^2 \quad)$$

f) $yy'' + (y')^2 = (y')^3$, baldin $y(1) = 1$ eta $y'(1) = 1$ badira.

$$(Em.: y(x) = x \quad)$$

7. Askatu bigarren ordenako ekuazio hauek:

a) $y'' - y = 0.$

(Em.: $y(x) = K_1 e^x + K_2 e^{-x}$)

b) $y'' + y = 0.$

(Em.: $y(x) = K_1 \sin x + K_2 \cos x$)

c) $y'' - 4y' + 2y = 0.$

(Em.: $y(x) = K_1 e^{(2+\sqrt{2})x} + K_2 e^{(2-\sqrt{2})x}$)

d) $y = y'' + y'.$

(Em.: $y(x) = K_1 e^{\left(\frac{-1-\sqrt{5}}{2}\right)x} + K_2 e^{\left(\frac{-1+\sqrt{5}}{2}\right)x}$)

e) $y'' = y/a^2$, baldin $y(0) = a$ eta $y'(0) = 0$ badira.

(Em.: $y(x) = \frac{a}{2} e^{x/a} + \frac{a}{2} e^{-x/a}$)

8. Askatu honako ekuazio diferentzial hauek:

a) $y''' + y = 0.$

(Em.: $y(x) = K_1 e^{-x} + K_2 e^{x/2} \sin \frac{\sqrt{3}x}{2} + K_3 e^{x/2} \cos \frac{\sqrt{3}x}{2}$)

b) $y^{iv} - 4y = 0.$

(Em.: $y(x) = K_1 e^{\sqrt{2}x} + K_2 e^{-\sqrt{2}x} + K_3 \sin \sqrt{2}x + K_4 \cos \sqrt{2}x$)

c) $y^{iv} + 8y'' + 16y = 0.$

(Em.: $y(x) = K_1 \sin(2x) + K_2 \cos(2x) + K_3 x \sin(2x) + K_4 x \cos(2x)$)

d) $y^{iv} + 2y''' + y'' = 0.$

(Em.: $y(x) = K_1 + K_2 x + K_3 e^{-x} + K_4 x e^{-x}$)

9. Aurkitu ekuazio hauen soluzio orokorra:

a) $2y'' - y' - y = 4xe^{2x}.$

(Em.: $y(x) = K_1 e^x + K_2 e^{-x/2} + e^{2x} \left(\frac{4}{5}x - \frac{28}{25} \right)$)

b) $y'' - 2y' + y = xe^x.$

(Em.: $y(x) = K_1 e^x + K_2 x e^x + \frac{1}{6} x^3 e^x$)

c) $y'' - y' + y = x^3 + 6.$

$$(Em.: y(x) = K_1 e^{x/2} \sin \frac{3x}{2} + K_2 e^{x/2} \cos \frac{3x}{2} + x^3 + 3x^2)$$

d) $y'' + y' = 5x + 2e^x.$

$$(Em.: y(x) = K_1 + K_2 e^{-x} + \frac{5}{2}x^2 - 5x + e^x)$$

e) $y'' + 10y' + 25y = 14e^{-5x}.$

$$(Em.: y(x) = K_1 e^{-5x} + K_2 x e^{-5x} + 7x^2 e^{-5x})$$

f) $y'' - 3y' + 2y = 14 \sin 2x - 18 \cos 2x.$

$$(Em.: y(x) = K_1 e^{2x} + K_2 e^x + 2 \sin 2x + 3 \cos 2x)$$

g) $y'' - 2y' + 5y = 25x^2 + 12.$

$$(Em.: y(x) = K_1 e^x \cos 2x + K_2 e^x \sin x + 2 + 4x + 5x^2)$$

h) $y'' + 4y = 4 \cos 2x + 6 \cos x + 8x^2 - 4x.$

$$(Em.: y(x) = K_1 \cos 2x + K_2 \sin 2x + x \sin 2x + 2 \cos x - 1 - x + 2x^2)$$